Quiz 2 (sample)

Name: ______________________________

Question 1. — Use the definition of the derivative to compute $f'\left(\frac{3}{2}\right)$, where $f(x) = \dfrac{(2x^3 - 7x^2 + 9)\sqrt{2x+1}}{\sin(\pi/x)}$. Simplify your answer as much as possible.

Question 2. — Use the definition of the derivative to compute $\dfrac{dy}{dx}$, where $y = \dfrac{x}{\sqrt{2x-3}}$. Simplify the result.

Quiz 2 (sample), with solutions

Question 1. — Use the definition of the derivative to compute $f'\left(\frac{3}{2}\right)$, where $f(x) = \dfrac{(2x^3 - 7x^2 + 9)\sqrt{2x+1}}{\sin(\pi/x)}$. Simplify your answer as much as possible.

Solution. — Notice that

$$2x^3 - 7x^2 + 9 = (x-3)(2x^2 - x - 3) = (x-3)(2x-3)(x+1), \qquad \text{so} \qquad y = 0 \quad \text{if} \quad x = \tfrac{3}{2}.$$

Then by the definition of the derivative,

$$\left.\frac{dy}{dx}\right|_{x=\frac{3}{2}} = \lim_{x\to\frac{3}{2}} \frac{y}{x-\frac{3}{2}} = \lim_{x\to\frac{3}{2}} \frac{(x-3)(2x-3)(x+1)\sqrt{2x+1}}{\left(x-\frac{3}{2}\right)\sin(\pi/x)} = \lim_{x\to\frac{3}{2}} \frac{2(x-3)(x+1)\sqrt{2x+1}}{\sin(\pi/x)} = \frac{2\cdot\left(-\frac{3}{2}\right)\cdot\left(\frac{5}{2}\right)\sqrt{4}}{\sin\left(\frac{2}{3}\pi\right)} = \frac{-15}{\frac{1}{2}\sqrt{3}} = -10\sqrt{3}.$$

Question 2. — Use the definition of the derivative to compute $\dfrac{dy}{dx}$, where $y = \dfrac{x}{\sqrt{2x-3}}$. Simplify the result.

Solution. — By the definition of the derivative,

$$\frac{dy}{dx} = \lim_{x'\to x} \frac{y'-y}{x'-x}, \qquad \text{where} \qquad y = \frac{x}{\sqrt{2x-3}} \qquad \text{and} \qquad y' = \frac{x'}{\sqrt{2x'-3}}.$$

Now

$$y' - y = \frac{x'\sqrt{2x-3} - x\sqrt{2x'-3}}{\sqrt{x'-3}\sqrt{x-3}} = \frac{x'^2(2x-3) - x^2(2x'-3)}{\sqrt{2x'-3}\sqrt{2x-3}\left(x'\sqrt{2x-3} + x\sqrt{2x'-3}\right)},$$

in which the numerator is $2x'x(x'-x) - 3(x'^2 - x^2) = (x'-x)\left(2x'x - 3(x'+x)\right)$. Therefore,

$$\frac{dy}{dx} = \lim_{x'\to x} \frac{2x'x - 3(x'+x)}{\sqrt{2x'-3}\sqrt{2x-3}\left(x'\sqrt{2x-3} + x\sqrt{2x'-3}\right)} = \frac{2x^2 - 6x}{(2x-3)\cdot 2x\sqrt{2x-3}} = \frac{x-3}{(2x-3)^{3/2}}.$$