Quiz 2 (sample)

Name: ______________________________

Question 1. — Evaluate the following integral.

$$\int \frac{\sec(\pi + \sqrt[3]{y^2})}{\sqrt[3]{y}}\, dy$$

Question 2. — Evaluate the following integral.

$$\int \frac{w-1}{e^w \sqrt{1 - w^2 e^{-2w}}}\, dw$$

Question 3. — Evaluate the following definite integral. ***Use the change of variables indicated and change the limits of integration.***

$$\int_0^{16} \frac{\sqrt{x}}{(1 + x^{3/4})^{3/2}}\, dx \qquad [\,\text{Let } t = \sqrt{1 + x^{3/4}}.\,]$$

Solutions

Question 1. — Evaluate the following integral.

$$\int \frac{\sec(\pi + \sqrt[3]{y^2})}{\sqrt[3]{y}}\, dy$$

Solution. — If $z = \pi + \sqrt[3]{y^2} = \pi + y^{2/3}$, then $dz = \frac{2}{3} y^{-1/3}\, dy$, or $\frac{3}{2}\, dz = dy/\sqrt[3]{y}$. Therefore,

$$\int \frac{\sec(\pi + \sqrt[3]{y^2})}{\sqrt[3]{y}}\, dy = \frac{3}{2} \int \sec(z)\, dz = \frac{3}{2} \ln|\sec(z) + \tan(z)| + C = \frac{3}{2} \ln\left|\sec(\pi + \sqrt[3]{y^2}) + \tan(\pi + \sqrt[3]{y^2})\right| + C.$$

Question 2. — Evaluate the following integral.

$$\int \frac{w-1}{e^w \sqrt{1 - w^2 e^{-2w}}}\, dw$$

Solution. — If $x = we^{-w}$ then $dx = (e^{-w} - we^{-w})\, dw = e^{-w}(1-w)\, dw$, so $-dx = \dfrac{w-1}{e^w}\, dw$. Therefore,

$$\int \frac{w-1}{e^w \sqrt{1 - w^2 e^{-2w}}}\, dw = -\int \frac{dx}{\sqrt{1-x^2}} = -\arcsin(x) + C = -\arcsin(we^{-w}) + C.$$

Question 3. — Evaluate the following definite integral. ***Use the change of variables indicated and change the limits of integration.***

$$\int_0^{16} \frac{\sqrt{x}}{(1 + x^{3/4})^{3/2}}\, dx \qquad [\text{ Let } t = \sqrt{1 + x^{3/4}}.\ ]$$

Solution. — If $t = \sqrt{1 + x^{3/4}}$ then $x = (t^2 - 1)^{4/3}$, $dx = \frac{8}{3} t(t^2-1)^{1/3}\, dt$, and $\sqrt{x} = (t^2-1)^{2/3}$. If $x = 0$, then $t = 1$, and if $x = 16$, then $t = 3$. Therefore,

$$\begin{aligned}
\int_0^{16} \frac{\sqrt{x}}{(1 + x^{3/4})^{3/2}}\, dx &= \frac{8}{3} \int_1^3 \frac{(t^2-1)^{2/3}}{t^3} \cdot t(t^2-1)^{1/3}\, dt = \frac{8}{3} \int_1^3 \frac{t^2 - 1}{t^2}\, dt = \frac{8}{3} \int_1^3 \left(1 - t^{-2}\right) dt \\
&= \frac{8}{3} \left(t + t^{-1}\right) \Big|_1^3 \\
&= \frac{8}{3} \left(2 - \frac{2}{3}\right) \\
&= \frac{32}{9}.
\end{aligned}$$