CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

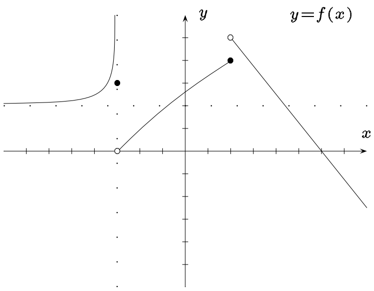
(Mathematics 201–NYA/∞)

1. Determine each of the following limits. If the limit does not exist state this and/or use ∞ or $-\infty$ as appropriate (show your work).

   (a) $\lim\limits_{x\to -1} \dfrac{x^2+3x+2}{x^2-x-2}$ (b) $\lim\limits_{x\to 2} \dfrac{\sqrt{6-x}-2}{2-x}$

   (c) $\lim\limits_{x\to 3^-} \dfrac{|x-3|}{(x-3)^2}$ (d) $\lim\limits_{x\to \infty} \dfrac{3x^3+4x+2}{5-x-x^3}$

2. Use the graph of $f(x)$ given below to find the limits. If a limit fails to exist, assign one of the symbols $+\infty$ or $-\infty$ if possible.



   (a) $\lim\limits_{x\to -\infty} f(x)$

   (b) $\lim\limits_{x\to 2^-} f(x)$

   (c) $\lim\limits_{x\to 2^+} f(x)$

   (d) $\lim\limits_{x\to 2} f(x)$

   (e) $\lim\limits_{x\to \infty} f(x)$

   (f) $\lim\limits_{x\to -3^+} f(x)$

3. Evaluate or estimate $\lim\limits_{x\to 0} \dfrac{\sin^2 x}{2x}$.

4. Use the limit definition of the derivative to show that if $f(x) = \dfrac{1}{1+x}$, then $f'(x) = -\dfrac{1}{(1+x)^2}$.

5. Use the definition of continuity to determine if
$$f(x) = \begin{cases} x-4 & \text{if } x \leqslant 2, \\ x^2-6 & \text{if } x > 2, \end{cases}$$ is continuous at $x=2$.

6. Find the value of k such that $g(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3, \\ k & \text{if } x = 3, \end{cases}$ is continuous at $x=3$.

7. Find the derivative of each of the following functions. *Do not simplify your answers.*

   (a) $y = 3x^2 - \dfrac{4}{\sqrt[3]{x}} + \log \pi$ (b) $f(x) = \dfrac{3x-5}{\sqrt{x}+1}$

   (c) $y = 5\sec^2(3x) + \tan(3x^2)$ (d) $g(x) = e^{-x^2}\ln(1+x)$

   (e) $y = \sqrt{x^2-a^2}$, where a is constant. (f) $y = \left(x^2+1\right)^{1/x}$

8. Let $f(x) = x^2e^{2x}$. Determine any values of x where the tangent line to the graph of $f(x)$ is horizontal.

9. Given the position function $s = t^{1/2} + t^{-1/2}$, find the velocity when the acceleration is zero.

10. Given $x^2+y^2 = 3y$, find an equation for the tangent line to the curve at $(-\sqrt{2}, 2)$.

11. Determine the absolute maximum and absolute minimum values of $f(x) = 12x - x^3$ on the interval $[0, 4]$.

12. For the function $f(x) = x^{2/3}(x+5)$, whose first and second derivatives are
$$f'(x) = \frac{5(x+2)}{3x^{1/3}} \quad \text{and} \quad f''(x) = \frac{10(x-1)}{9x^{4/3}}.$$

    (a) Find the intervals of increase/decrease.
    (b) Find the intervals of concavity.
    (c) Find the coordinates of all relative (or local) extreme points.
    (d) Find the coordinates of all points of inflection.
    (e) Sketch the graph.

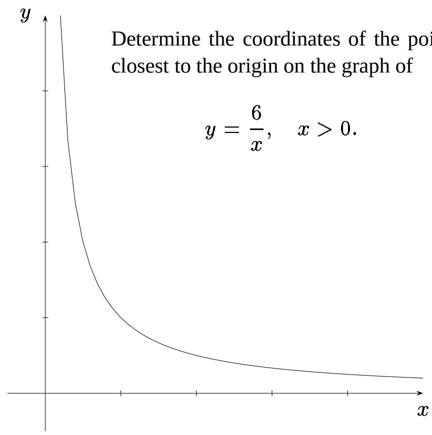
    Make sure that your graph clearly illustrates all these features.

13. Given $V = \frac{4}{3}\pi r^3$ (the volume of a sphere) and $S = 4\pi r^2$ (the surface area of a sphere), consider the following problem.

    The volume of a large spherical balloon is decreasing at a constant rate of 50 m^3/min.

    (a) How fast is the radius r of the balloon decreasing at the instant the radius is 4 metres?
    (b) How fast is the surface area S decreasing at the instant the radius is 4 metres?

14. Determine the coordinates of the point closest to the origin on the graph of
$$y = \frac{6}{x}, \quad x > 0.$$



15. Evaluate the following integrals:

    (a) $\displaystyle\int \left(2x - 1 + \frac{3}{x} - \frac{4}{x^2}\right)\,dx$ (b) $\displaystyle\int_{-\pi/6}^{\pi/6} \tfrac{1}{2}\cos x\;dx$

    (c) $\displaystyle\int \left(e^x + x^e\right)\,dx$ (d) $\displaystyle\int_1^2 3x\left(x^2+5\right)\,dx$

16. Find the area of the region enclosed by $y = x^2 + \sec^2 x$ and $y = 0$ from $x = 0$ to $x = \frac{\pi}{4}$.