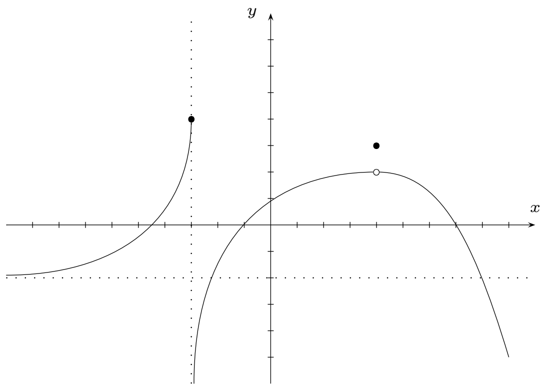
CALCULUS I (SCIENCE)

(MATHEMATICS 201–NYA/∞)

1. (a) Refer to the accompanying graph of $y = f(x)$ in order to evaluate the following. Use the terms ∞, $-\infty$, and *does not exist*, as appropriate.



(i) $\displaystyle\lim_{x\to-\infty} f(x)$ (ii) $\displaystyle\lim_{x\to-3^+} f(x)$ (iii) $\displaystyle\lim_{x\to4^-} f(x)$

(b) Show that $f(x)$ is discontinuous at $x = 4$.

(c) Classify the discontinuity at $x = 4$.

2. Determine each of the following limits. Use the terms ∞, $-\infty$, and *does not exist*, as appropriate. Include adequate justification and use correct mathematical notation.

(a) $\displaystyle\lim_{x\to0} \frac{\sqrt{x+4}-2}{x}$ (b) $\displaystyle\lim_{x\to2^+} \frac{x^2-5x+6}{x^2-4x+4}$

(c) $\displaystyle\lim_{x\to3} \frac{(x-8)^2-25}{3-x}$ (d) $\displaystyle\lim_{x\to-\infty} \frac{3+9x^2}{1+3x}$

3. Let $g(t) = \begin{cases} 3t+2 & \text{if } t \leqslant 1 \\ 5 & \text{if } 1 < t \leqslant 3 \\ 3t^2-1 & \text{if } t > 3 \end{cases}$.

Determine all points of discontinuity of the function $g(t)$. Justify your answers.

4. The position of a particle moving in a straight line is given by $s = t^4 - 8t^3 + 16t^2 + 2$ where t is measured in seconds and s in metres.

(a) Show that the particle is at rest when $t = 4$.

(b) Find the acceleration at $t = 4$.

5. (a) Use the limit definition of derivative to find $f'(x)$ for $f(x) = \dfrac{2}{x+1}$.

(b) Find the derivatives of the following functions. Do not simplify.

(i) $y = 5x^{10} + \dfrac{8}{\sqrt[4]{x}} + e^{\csc x}$ (ii) $y = x^5 \ln\left(x^3+1\right)$

(iii) $y = \dfrac{x^3+2x}{x^7-3}$ (iv) $y = \cos^2(Ax+B)$

(v) $y = (1+x)^{\sin x}$

6. Find the second derivative of the function $f(x) = \sec x \tan x$.

7. Compute $\dfrac{dy}{dx}$ for $x + \ln(xy) = 2$. Find an equation for the tangent line to the curve at the point $(1, e)$.

8. At what values of x does $f(x) = (5x-2)^4(1-x)^6$ have a horizontal tangent line?

9. The infield of a 400 metre track consists of a rectangle with semicircular ends.

To what dimensions should the track be built in order to maximize the area of the rectangle?

10. Find the extreme values (*i.e.*, the absolute maximum and absolute minimum) of $p(x) = 4x^3 + 6x^2 - 72x + 13$ on the interval $[-2, 3]$.

11. The area of a square is decreasing at a rate of $10\text{m}^2/\text{h}$. How fast is the diagonal of the square decreasing when the length of a side is 5 metres?

12. Find all x- and y-intercepts, vertical and horizontal asymptotes, local extrema, point(s) of inflection, intervals of increasing/decreasing and intervals of concavity. Then sketch the graph of

$$f(x) = \frac{2+x-x^2}{(x-1)^2}, \text{ given: } f'(x) = \frac{x-5}{(x-1)^3};\ f''(x) = \frac{2(7-x)}{(x-1)^4}.$$

13. Evaluate the following integral by interpreting it in terms of area:

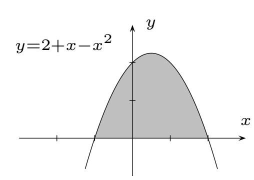
$$\int_{-5}^{0} \sqrt{25-x^2}\,dx.$$

14. Find:

(a) $\displaystyle\int \frac{x^7}{3} + \frac{4}{x^2} - 5\sqrt[3]{x^2} + 6\,dx$ (b) $\displaystyle\int \frac{2\sin x - 5}{\cos^2 x}\,dx$

(c) $\displaystyle\int \frac{e^x}{3} + \frac{5}{x} - 8x^{1/2}\,dx$

15. Find the area between the curve $y = 2 + x - x^2$ and the x-axis.



16. $f''(x) = 3\cos x + 2\sin x$, $f'(0) = 3$ and $f(0) = 2$. Solve for $f(x)$.