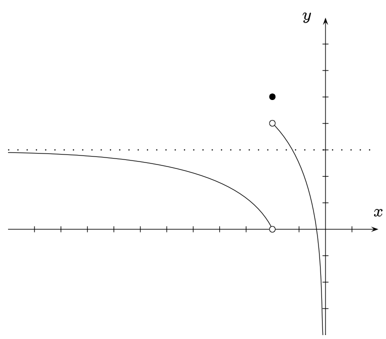
CEGEP JOHN ABBOTT COLLEGE

CALCULUS I (SCIENCE)
(MATHEMATICS 201-NYA/∞)

1. Given the graph below, evaluate
   (a) $\lim\limits_{x\to -2^+} f(x)$ (b) $\lim\limits_{x\to 0^-} f(x)$ (c) $\lim\limits_{x\to -2} f(x)$
   (d) $\lim\limits_{x\to -\infty} f(x)$ (e) $f(-2)$



(Each mark on an axis represents one unit.)

2. Kindly evaluate the following limits, showing all relevant steps. Use the symbols $+\infty$ and $-\infty$ where appropriate.

   (a) $\lim\limits_{x\to 7} \dfrac{x^2-6x-7}{x^2-2x-35}$ (b) $\lim\limits_{x\to \frac{\pi}{6}} \dfrac{\tan 2x}{x}$

   (c) $\lim\limits_{x\to -\infty} \dfrac{5-3x+5x^2}{12+4x-3x^2}$ (d) $\lim\limits_{x\to 3} \dfrac{x^2-4x+3}{x^2-6x+9}$

3. Let $f(x) = \begin{cases} kx+1 & \text{if } x<3 \\ kx^2-1 & \text{if } x \geqslant 3 \end{cases}$.

   Determine all values of k (if there are any) for which f is continuous at the number 3.

4. Sketch the graph of a function that satisfies the conditions:

$$\lim_{x\to 5} f(x) = 3, \quad f(5) = 4, \quad \text{and} \quad \lim_{x\to 8} f(x) = -\infty.$$

5. Use the limit definition of derivative to find $f'(x)$ where $f(x) = \dfrac{1}{x+2}$.

6. Find the derivatives of the following functions. *Do not simplify.*

   (a) $y = 5x^{8/5} + \dfrac{4}{\sqrt[3]{x}} + e^{3\ln x} + \tan(e^x)$ (b) $y = \sqrt{x^2+1}\,\sec(3x)$

   (c) $y = \dfrac{x^3+2}{x^5-7}$ (d) $y = \sin^3(\ln x)$

   (e) $y = e^{x^4}\cos(5x)$ (f) $y = (\sin x)^{x^2+3}$

7. If a ball is thrown vertically upward with a velocity of 128 feet/sec, then its height h after t seconds is given by $h = 128t - 16t^2$.
   (a) What is the maximum height reached by the ball?
   (b) What is the velocity of the ball when it is 192 feet above the ground?

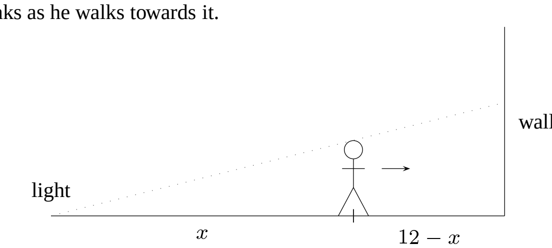
8. Let $y = \frac{1}{2}x^2 + \ln x$. Find all x at which $y'' = 0$.

9. Compute $\dfrac{dy}{dx}$ from the equation $x^2 + 2xy + y^3 = 4$ at $(1, 1)$. Also find an equation of the tangent line at $(-2, 0)$.

10. Let $f(x) = x^3 - 3x + 1$, $0 \leqslant x \leqslant 3$. [Assume that] f is continuous throughout the closed, bounded interval $[0, 3]$. According to the extreme value theorem, f must attain an absolute maximum and an absolute minimum somewhere on that interval.
    Your job: Find them (And tell where they are attained).

11. For the following problem: (a) produce an explicit function depending on one variable only whose optimization solves the problem; (b) state what the variable represents and any restrictions on it. *Do not solve the problem.*
    A closed box with a volume of 23.04 cubic units is to be constructed. Its base is to be 3 times as long as it is wide. If the material for the base costs \$10.50 per square unit and that for the remaining pieces costs \$7 per square unit, what are the dimensions of the cheapest such box?

12. A spotlight on the ground shines on a [very tall vertical] wall 12 metres away. A man 2 metres tall walks directly from the spotlight towards the wall at a constant speed of 1.6 metres per second. His shadow on the wall shrinks as he walks towards it.



How fast is his shadow shrinking at the instant he is 4 metres from the wall?

13. Let $g(x) = -\dfrac{27(x-2)^2}{(x-3)^3}$. Then

$$g'(x) = \frac{27x(x-2)}{(x-3)^4} \quad \text{and} \quad g''(x) = -\frac{54(x^2-3)}{(x-3)^5}.$$

Also, $g(-\sqrt{3}) \approx 3.549$ and $g(\sqrt{3}) \approx 0.95096$. Sketch a graph of g including all asymptotes, intercepts, local extrema and points of inflection labelled as such.

14. Determine all values of x for which the derivative of the given function is zero.

$$y = x\sqrt{18-x^2}, \quad -3\sqrt{2} \leqslant x \leqslant 3\sqrt{2}$$

15. Find $f(x)$ if $f'(x) = 5 - x^{-1/2}$ and $f(4) = 13$.

16. Evaluate:
    (a) $\displaystyle\int 3e^x + \frac{5}{2x} - \sqrt[3]{x^2}\,dx$ (b) $\displaystyle\int \sin x\,(\cot x - 1)\,dx$

    (c) $\displaystyle\int \frac{5x^4 - 4x^3 + 6}{2x^3}\,dx$ (d) $\displaystyle\int_{-1}^{2} 3x^2 - 2x - 5\,dx$

    (e) $\displaystyle\int_{-3}^{2} |x^2 - 4|\,dx$