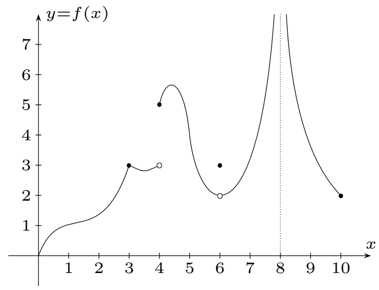
1. Below is the graph of a function $y = f(x)$ with domain contained in the closed interval $[0, 10]$. Open circles represent points not on the graph of f and solid circles represent points that are on the graph.
   (a) For what values of x does the function have
   (i) jump,
   (ii) infinite,
   (iii) removable,
   discontinuities?
   (b) Determine
   (i) $\lim_{x\to 4^-} f(x)$,
   (ii) $\lim_{x\to 4^+} f(x)$,
   (iii) $\lim_{x\to 4} f(x)$,
   (iv) $\lim_{x\to 8} f(x)$.



2. Calculate the following limits if possible. Note that a limit is allowed to be ∞ or $-\infty$. If a limit does not exist, explain why it does not exist, and include the left and right limits in your answer as appropriate.

   (a) $\displaystyle\lim_{x\to -2} \frac{2x^2+x-6}{x+2}$ (b) $\displaystyle\lim_{x\to 3^-} \frac{x^2+1}{x^2-9}$

   (c) $\displaystyle\lim_{x\to \infty} \frac{x+3}{\sqrt{x^2+1}}$ (d) $\displaystyle\lim_{x\to 2^-} \left(\frac{1}{x-2} - \frac{4}{x^2-4}\right)$

3. Find the derivative of the function $f(x) = \sqrt{2x}$ using the *limit* definition of the derivative.

4. Find the derivative of each of the following functions. *Do not simplify your answers*.

   (a) $y = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}} - \sin\sqrt{x}$ (b) $y = \dfrac{e^{5x}}{5x+e^{5x}} + e^{5\pi}$

   (c) $y = \csc x + e^{\cos x}\cos x$ (d) $f(x) = \ln(x^3+3x)$

   (e) $f(x) = \sec(\tan 3x)$ (f) $f(x) = (x^2+1)^{\sin x}$

5. The curve $y(x^2+1) = 1$ is called a *witch of Maria Agnesi*. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.

6. Give the second derivative of the function $h(t) = e^t \sin t$.

7. Find $\dfrac{dy}{dx}$ by *implicit differentiation* for $\dfrac{x+3}{y} = 4x + y^2$.

8. A particle moves along a straight line with equation of motion
$$s = t^3 - 9t^2 + 15t + 10,$$
   where s is measured in metres and t is measured in seconds.
   (a) Find the acceleration when $t = 2$.
   (b) When is the particle at rest?
   (c) When is the particle moving in the positive direction?
   (d) When does the particle reach a velocity of -6 m/s?

9. Suppose a forest fire spreads in a circle with the radius changing at the rate of 2 metres per minute. When the radius reaches 80 metres, at what rate is the area of the burning region increasing?

10. Let $f(x) = x^3+x^2-8x-1$, for $-3 \leqslant x \leqslant 4$. Find the absolute maximum value and the absolute minimum value of f and state where they are attained.

11. Consider the following problem:
   An open rectangular box with a square base must have a volume of $13{,}500$ cm^3. Find the dimensions of the box that minimize the amount of material used.
   (a) Produce an explicit function depending on one variable only whose optimization solves the problem.
   (b) State what the variable represents and any restrictions on it. (c) STOP

12. Let $f(x) = \dfrac{(2x+3)(x-3)^2}{x^3}$. Then
$$f'(x) = 9\frac{(x-3)(x+3)}{x^4} \quad \text{and} \quad f''(x) = -18\frac{x^2-18}{x^5}.$$
   Sketch a graph of f including all asymptotes, incercepts, local extrema and points of inflection.

13. Find $f(x)$ given $f'(x) = e^x - 2\sin x$, and $f(0) = -3$.

14. Evaluate the following integrals.
   For part (d) give the exact answer, no decimals.

   (a) $\displaystyle\int \left(5x^2 - \frac{5}{x^2} - \sqrt[5]{x^2} - 2^5\right)dx$ (b) $\displaystyle\int \frac{\cos x}{1-\cos^2 x}\,dx$

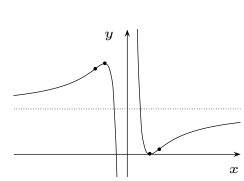
   (c) $\displaystyle\int \frac{(x-3)^2}{x}\,dx$ (d) $\displaystyle\int_{\pi/6}^{\pi/3} (12 - \sec^2 x)\,dx$

15. FInd the area of the region between $y = 4x + x^2$ and the x-axis from $x = 1$ to $x = 3$.

16. Evaluate $\displaystyle\int_{-1}^{4} |x-3|\,dx$ by interpreting it in terms of areas.

ANSWERS

1. (a) (i) 4, (ii) 8, (iii) 6; (b) (i) 3, (ii) 5, (iii) the limit DNE, (iv) ∞.
2. (a) -7, (b) $-\infty$, (c) 1, (d) $\frac{1}{4}$.
3. $f'(x) = \lim_{h\to 0} \left\{\left(\sqrt{2(x+h)} - \sqrt{2x}\right)/h\right\} = 1/\sqrt{2x}$
4. (a) $\frac{3}{2}x^{1/2} - \frac{5}{2}x^{-7/2} - (\cos\sqrt{x})/(2\sqrt{x})$,
   (b) $\dfrac{5e^{5x}(5x+e^{5x}) - e^{5x}(5+5e^{5x})}{(5x+e^{5x})^2}$,
   (c) $-\csc x\cot x - e^{\cos x}\sin x(\cos x + 1)$, (d) $\dfrac{3x^2+3}{x^3+3x}$,
   (e) $3\sec(\tan 3x)\tan(\tan 3x)\sec^2 3x$,
   (f) $(x^2+1)^{\sin x}\left\{\cos x\ln(x^2+1) + \dfrac{2x\sin x}{x^2+1}\right\}$
5. $x - 2y + 2 = 0$ 6. $h''(t) = 2e^t\cos t$ 7. $\dfrac{y-4y^2}{x+3+2y^3}$
8. (a) -6 m/s^2, (b) $t = 1$ s or 5 s, (c) $t < 1$ s and $t > 5$ s, (d) $t = 3 \pm \sqrt{2}$ s.
9. 320π m^2/s
10. On $[-3, 4]$ f attains its maximum value, 47, at the endpoint $x = 4$, and f attains its minimum value, $-203/27$, at the critical number $x = 4/3$.
11. (a) $A(x) = x^2 + 54000/x$, where (b) x is the length of a side of the base of the box, which must be positive.
12.



Intercepts: $(-\frac{3}{2}, 0)$; $(3, 0)$.

Asymptotes: HA: $y = 2$; VA: $x = 0$.

Extrema: $(-3, 4)$ (local max); $(3, 0)$ (local min). There are no global extrema.

IP's: $(-3\sqrt{2}, 2 + \frac{5}{4}\sqrt{2})$; $(3\sqrt{2}, 2 - \frac{5}{4}\sqrt{2})$.

13. $f(x) = e^x + 2\cos x - 6$.
14. (a) $\frac{5}{3}x^3 + 5/x - \frac{5}{7}x^{7/5} - 32x + C$, (b) $-\csc x + C$
   (c) $\frac{1}{2}x^2 - 6x + 9\ln|x| + C$, (d) $2\pi - \frac{2}{3}\sqrt{3}$.
15. $74/3$ 16. $17/2$