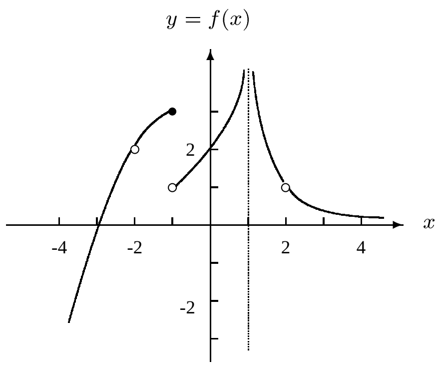
# CALCULUS I (SCIENCE)

(MATHEMATICS 201–NYA/$\infty$)

1. By referring to the graph of $f(x)$, determine the following limits.

(a) $\lim_{x\to -3} f(x)$
(b) $\lim_{x\to -1^+} f(x)$
(c) $\lim_{x\to -1} f(x)$
(d) $\lim_{x\to -2} f(x)$
(e) $\lim_{x\to 2^-} f(x)$
(f) $\lim_{x\to 1^-} f(x)$
(g) $\lim_{x\to -\infty} f(x)$




2. Evaluate the following limits. Note that the limit may be $\infty$ or $-\infty$. State DNE (does not exist) if the limit is neither $\pm\infty$ nor a real number.

(a) $\displaystyle\lim_{x\to\sqrt{5}^-} \frac{12x-5}{x^2-5}$ (b) $\displaystyle\lim_{x\to e} \left(\pi^{x-e} + 2\ln x\right)$

(c) $\displaystyle\lim_{x\to 3} \frac{x^2+x-12}{2x^2-5x-3}$ (d) $\displaystyle\lim_{x\to 2} \frac{\frac{1}{x+1}-\frac{1}{3}}{x^2-4}$ (e) $\displaystyle\lim_{x\to -\infty} \frac{5x^3+2}{x^2-3}$

3. State the definition of the derivative as a limit, and use this definition to find the derivative of $f(x) = \sqrt{x-5}$.

4. (a) State the three conditions for a function $f(x)$ to be continuous at $x = c$.
(b) State any discontinuities of the following function. *Justify your answer*.

$$f(x) = \begin{cases} \dfrac{1}{x+1} & \text{if } x < 0 \text{ and } x \neq -1, \\ x^2 & \text{if } 0 \leqslant x \leqslant 2, \\ 4-x & \text{if } x > 2. \end{cases}$$

5. Find $\dfrac{dy}{dx}$ for each of the following. *Do not simplify your answers*.

(a) $y = 4x^3 - \dfrac{3}{x^4} + \dfrac{1}{\sqrt[3]{x^4}} + 3^4$ (b) $y = \dfrac{e^x+1}{e^{2x}+2}$

(c) $y = 4^x \ln(x^2-4)$ (d) $y = (1+x^2)^{\sqrt{x}}$

(e) $y = \dfrac{x^4\sqrt[5]{1+x^2}}{\sin^3 x}$ (f) $y = \tan^4\left(e^{-3x^2}\right)$

6. Use implicit differentiation to find the slope of the tangent line to the curve $x^2 + 4y^2 - 7x + 6 = 0$ at the point $(2, -1)$.

7. The equation of motion of a particle moving along a straight line is

$$s = t^3 - 4t^2 + 4t + 11,$$

where $s$ is measured in metres and $t$ is measured in seconds.
(a) Find the velocity at time $t$. (b) When is the particle at rest?

8. Let $\triangle ABC$ be a right triangle with hypotenuse $AC$, whose base $AB$ is 7 cm long. The height $BC$ is increasing in length at a rate of 2 cm/s. When $BC = 24$ cm, at what rate is the lenght of the hypotenuse changing?

9. Find the absolute maximum and the absolute minimum values of

$$f(x) = x^3 + 3x^2 - 24x + 3$$

on the interval $[0, 4]$.

10. Given $f(x) = 12 + 2x^2 - x^4$. (a) Determine all regions where $f$ is increasing, decreasing, concave up and concave down. (b) Sketch the graph of $f$. Show all intercepts, asymptotes (with equations), local extrema and points of inflection, *if any*.

11. Find the point on the graph of $2y = x^2$, closest to the point $(32, 1)$.

12. Determine the area under the graph of $y = x + 4\cos x$ between $x = 0$ and $x = \frac{1}{2}\pi$.

13. Evaluate $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx$ by considering the area under the graph of the function. *Your answer should include the appropriate picture.*

14. Find $S(t)$ given $S''(t) = 30\sqrt{t}$, $S(4) = 200$, and $S'(1) = 10$.

15. Evaluate the following integrals.

(a) $\displaystyle\int \left(4x^3 - \frac{4}{x^3} + \sqrt[4]{x^3} - \pi e^x + \ln\pi\right) dx$ (b) $\displaystyle\int \frac{(x+2)^2}{x^2}\,dx$

(c) $\displaystyle\int_{-1}^{1} (x^2 - e^x)\,dx$ (d) $\displaystyle\int_1^4 \frac{2x+3}{\sqrt{x}}\,dx$

## ANSWERS

1. (a) 0; (b) 1; (c) DNE; (d) 2; (e) 1; (f) $\infty$; (g) $-\infty$.
2. (a) $-\infty$; (b) 3; (c) 1; (d) $-1/36$; (e) $-\infty$.
3. $f'(x) = \displaystyle\lim_{h\to 0} \frac{\sqrt{(x+h)-5}-\sqrt{x-5}}{h} = \cdots = \frac{1}{2\sqrt{x-5}}$.
4. (a) $f$ is continuous at $c$ if $f(c) = \lim_{x\to c} f(x)$ (that's the def$^{\mathrm{n}}$; the question seems to be asking for some BS recipe); (b) $f$ is discontinuous at $-1$, 0 and 2.
5. (a) $12x^2 + 12x^{-5} - \frac{4}{3}x^{-7/3}$; (b) $\dfrac{e^x(e^{2x}+2) - 2e^{2x}(e^x+1)}{(e^{2x}+2)^2}$;

(c) $4^x(\ln 4)\ln(x^2-4) + \dfrac{4^x\,2x}{x^2-4}$; (d) $(1+x^2)^{\sqrt{x}}\left\{\dfrac{\ln(1+x^2)}{2\sqrt{x}} + \dfrac{2x\sqrt{x}}{1+x^2}\right\}$;

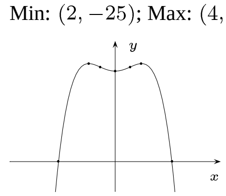
(e) $\dfrac{x^4\sqrt[5]{1+x^2}}{\sin^3 x}\left\{\dfrac{4}{x} + \dfrac{2x}{5(1+x^2)} - 3\cot x\right\}$;

(f) $4\tan^3\left(e^{-3x^2}\right)\sec^2\left(e^{-3x^2}\right)\left(-6xe^{-3x^2}\right)$;

6. $-\frac{3}{8}$ 7. (a) $v = 3t^2 - 8t + 4$; (b) $t = \frac{2}{3}$, 2 s. 8. $\frac{48}{25}$

9. Min: $(2, -25)$; Max: $(4, 19)$

10.




Intercepts: $(\pm\sqrt{1+\sqrt{13}}, 0)$.

Extrema: $(\pm 1, 13)$ (local max'a); $(0, 12)$ (local min). There are no global extrema.

IP's: $(\pm\frac{1}{3}\sqrt{3}, \frac{113}{9})$.

11. $(4, 8)$ 12. $\frac{1}{8}(\pi^2 + 32)$ 13. $4\pi$
14. $S(t) = 8t^{5/2} - 10t - 16$
15. (a) $x^4 + 2x^{-2} + \frac{4}{7}x^{7/4} - \pi e^x + x\ln\pi + C$; (b) $x + 4\ln|x| - 4/x + C$; (c) $\frac{2}{3} - e + 1/e$; (d) $46/3$