CALCULUS I (SCIENCE)

(MATHEMATICS 201–NYA/∞)

1. Evaluate the limits. Use the symbols $-\infty$ and ∞ where appropriate.

   (a) $\displaystyle\lim_{x\to-2}\frac{3x^2+5x-2}{(2x-1)(x+2)}$ (b) $\displaystyle\lim_{x\to1}\frac{\sqrt{3x+1}-2}{1-x^2}$

   (c) $\displaystyle\lim_{x\to-\infty}\frac{5-2x^2+x^3}{2+3x-x^2}$ (d) $\displaystyle\lim_{x\to0^-}\frac{\tan x-2\cos x+x/|x|}{2+\sin x-e^x}$

2. Use the limit definition of the derivative to find $f'(x)$ if $f(x)=\sqrt{2x+1}$.

3. Give a formula for, and draw the graph of, a function which is continuous on the interval $[-1,1]$ but is not differentiable at 0.

4. Find the derivative of each of the following.

   (a) $y=-4x^{-3/4}+2e^x-3\sin x+x^e$ (b) $y=\ln(x+\ln x)$

   (c) $y=\pi^x+\dfrac{x}{\pi}+x^\pi+\dfrac{\pi}{x}+\pi$ (d) $y=\tan^3\left(\dfrac{x+2}{x-2}\right)$

   (e) $y=(\cos x)^{(x^2+3)}$ (f) $y=e^{-\sqrt{x}}\sec(x\sin x)$

5. Find all critical numbers of the function $f(x)=e^{\sin x}(2\sin x-3)$.

6. You are given a function f which is differntiable on $\mathbb{R}$ and satisfies both $f(\sqrt{3})=6$ and $f'(\sqrt{3})=9$. Find an equation of the tangent line to the graph of $g(x)=f\big(\tan(\pi/x)\big)$ at the point where $x=3$.

7. Consider the curve given by $xy^2-x^3y=6$.

   (a) Show that $\dfrac{dy}{dx}=\dfrac{3x^2y-y^2}{2xy-x^3}$.

   (b) Find all point on the curve whose x-coordinate is 1, and write an equation of the tangent line at each of these points.

8. Given $y=\ln(\tan x+\sec x)$, find $\dfrac{d^3y}{dx^3}$ and express your answer in terms of $\sec x$.

9. Find all values of c, if any, which would make f continuous everywhere if

$$f(x)=\begin{cases}2(cx+1) & \text{if } x\leqslant-1,\\ -c^2x-1 & \text{if} > -1.\end{cases}$$

10. A skateboarder travelling east at 12 ft/sec just misses a squirrel which is running north at 9 ft/sec. At what rate is the distance between the skateboarder and the squirrel increasing 6 seconds after they have passed each other.

11. Determine the global maximum and minimum values of the function $f(x)=3x^4-4x^3$ on the interval $[-1,2]$.

12. The following picture shows the graph of a function f. Anwer the questions below by estimating the appropriate quantities based on this picture. (Each tick on an axis represents one unit.)

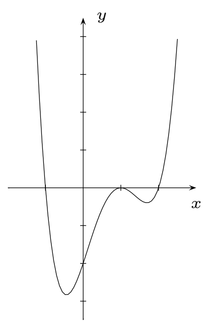
    (a) Find the value(s) of x where $f(x)$ changes sign.

    (b) Find the value(s) of x where $f(x)$ has a local maximum

    (c) Find the value(s) of x where $f(x)$ has a local minimum

    (d) Sketch the graph of $f(x)$ on the interval $[-1.2, 2.5]$.

    (e) Find the value(s) where $f''(x)$ changes sign.



13. If $f(x)=\dfrac{x^2-1}{x^3}$, then $f'(x)=\dfrac{3-x^2}{x^4}$ and $f''(x)=\dfrac{2(x^2-6)}{x^5}$.

    (a) Find the coordinates $(a, f(a))$ of all local extrema.

    (b) Find the intervals of downward concavity.

    (c) find the coordinates $(a, f(a))$ of all inflection points.

    (d) Find all vertical and horizontal asymptotes.

    (e) Sketch a graph of $y=f(x)$ for that part of the domain of f which is within the closed interval $[-4,4]$. Show all intercepts, local extrema, points of inflection and asymptotes.

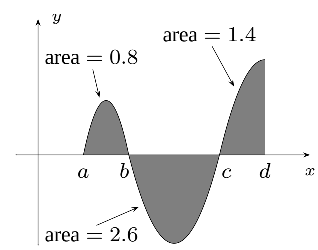
14. A beam of wood with a rectangular cross-section is to be cut from a cylindrical log. The formula used in engineering to calculate the strength s of such a beam is $S=kxy^2$ where k is a constant, x is the width of the beam and y is the depth of the beam. Find the dimensions of the strongest wooden beam which can be cut from a cylindrical log of diameter 30 cm.

15. Find the function $f(x)$ given $f''(x)=2x^3+3x^2-4x+5$, $f(0)=2$ and $f(1)=0$.

16. Use the areas in the figure to find the definite integrals.

    (a) $\displaystyle\int_a^c f(x)\,dx$

    (b) $\displaystyle\int_d^b f(x)\,dx$



17. Use the Fundamental Theorem of Calculus to find the derivative:

$$\frac{d}{dx}\int_1^x \sin(t^2)\,dt.$$

18. Compute the following indefinite integrals.

    (a) $\displaystyle\int\frac{\sec x+\cos x}{\cos x}\,dx$ (b) $\displaystyle\int x^{1/3}(2-x)^2\,dx$

    (c) $\displaystyle\int(2-3x+1/x-\sin x)\,dx$

19. Find the following definite integrals.

    (a) $\displaystyle\int_{-\frac14\pi}^{\frac14\pi}(1+2\cos x)\,dx$ (b) $\displaystyle\int_4^9(4y^{-1/2}+2y^{1/2}+y^{-5/2})\,dy$

20. Make a sketch of the curve $y=x^2+1$ over the interval $[0,3]$ and find the area of this region.

CEGEP
JOHN ABBOTT
COLLEGE

Calculus I (Science)

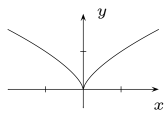
(Mathematics 201–NYA/∞)

Answers

1. (a) $\frac{7}{5}$ (b) $-\frac{3}{8}$ (c) ∞ (d) -3

2. $(2x+1)^{-1/2}$

3. One possible answer is $f(x) = x^{2/3}$.



4. (a) $\dfrac{dy}{dx} = 3x^{-7/4} + 2e^x - 3\cos x + ex^{e-1}$
   (b) $\dfrac{dy}{dx} = \dfrac{x+1}{x(x+\ln x)}$
   (c) $\dfrac{dy}{dx} = (\ln \pi)\pi^x + \pi^{-1} + \pi x^{\pi-1} - \pi x^{-2}$
   (d) $\dfrac{dy}{dx} = \dfrac{-12}{(x-2)^2}\tan^2\left(\dfrac{x+2}{x-2}\right)\sec^2\left(\dfrac{x+2}{x-2}\right)$
   (e) $\dfrac{dy}{dx} = (\cos x)^{(x^2+3)}(2x\ln\cos x - (x^2+3)\tan x)$
   (f) $\dfrac{dy}{dx} = e^{\sqrt{x}}\sec(x\sin x)\left\{(\sin x + x\cos x)\tan(x\sin x) - \frac{1}{2}x^{-1/2}\right\}$

5. $\frac{1}{6}\pi(12n+3\pm 2)$, $\frac{1}{2}\pi(2n+1)$, where $n \in \mathbb{Z}$.

6. $4\pi x + y = 6(2\pi + 1)$

7. (a) Differentiate the given equation with respect to x (and don't forget to apply the chain rule when you differentiate an expression that depends on y).
   (b) The points are $(1,-2)$ and $(1,3)$; the equations of the tangent lines are, respectively, $2x - y = 4$ and $y = 3$.

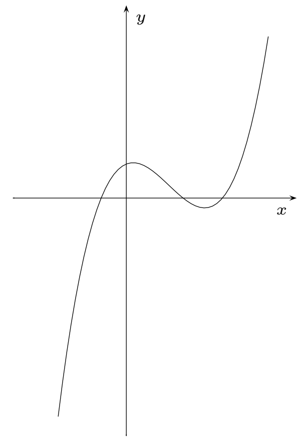
8. $\dfrac{d^3y}{dx^3} = \sec x(2\sec^2 x - 1)$.

9. $c = -3, 1$

10. After 6 seconds the distance between them is increasing by rate of 15 feet per second.

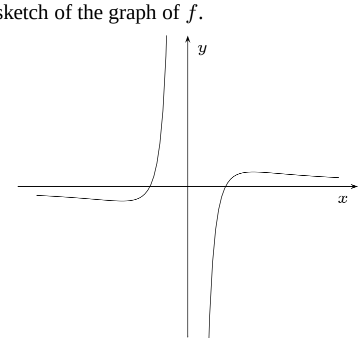
11. The global maximum is $f(2) = 16$ and the global minimum is $f(1) = -1$.

12. *Note*: It is a stupid and pointless waste of time to try to guess numerical values of a variable by looking at a graph, unless the marks on a graph communicate unambiguously what those values are. For this reason we give open intervals containing the relevant numerical values. We also give exact values, which you can obtain by using the equation whose graph is displayed: $y = (x-1)^2(x+1)(x-2)$.
   (a) One value is 1, and there a value in each of the intervals $(-1, 0)$ and $(1, 2)$. The exact values of the latter numbers are $(5 \pm \sqrt{73})/8$.
   (b) 1
   (c) There is a value in each of $(-1, 0)$ and $(1, 2)$. Their exact values are given in the answer to part (a).
   (d) Here is a graph of f'. It is not to scale; the y-axis is compressed by a factor of 5 compared to the x-axis. Again, it is a pointless waste of time to attempt to sketch the graph of f' with any real accuracy. Your sketch should reflect: (i) that f' is positive where f is increasing and f' is negative where f is decreasing, and (ii) that f' is increasing where the graph of f is concave up and f' is decreasing where the graph of f is concave down. Of course, your sketch can reflect these only roughly.



   (e) There is a value in each of the intervals $(0, 1)$ and $(1, 2)$; their exact values are $(9 \pm \sqrt{57})/12$.

13. (a) There is a local minimum at $(-\sqrt{3}, -\frac{2}{9}\sqrt{3})$ and a local maximum at $(-\sqrt{3}, -\frac{2}{9}\sqrt{3})$.
   (b) The graph of f is concave down on $(-\infty, -\sqrt{6})$ and $(\sqrt{6}, \infty)$.
   (c) The graph has inflection points at $(-\sqrt{6}, -\frac{5}{36}\sqrt{6})$ and $(\sqrt{6}, \frac{5}{36}\sqrt{6})$.
   (d) The vertical asymptote has equation $x = 0$ and the horizontal asymptote has equation $y = 0$.
   (e) Here is a sketch of the graph of f.



14. The strongest beam is $10\sqrt{3}$ cm wide and $10\sqrt{6}$ cm deep.

15. $f(x) = \frac{1}{10}x^5 + \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - \frac{251}{60}x + 2$.

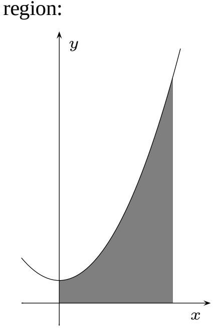
16. (a) -1.8 (b) 1.2

17. $\sin(x^2)$

18. (a) $\tan x + x + C$
    (b) $4x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C$

19. (a) $\frac{1}{2}(\pi + 4\sqrt{2})$
    (b) $10819/324$

20. Here is a sketch of the region:



The area of the region is 12 (square units).