1. Evaluate the limit or explain why it does not exist. Use ∞, $-\infty$ or "does not exist" where appropriate.

a. $\displaystyle\lim_{x\to -3}\frac{x^2+x-6}{x^3+3x^2-5x-15}$ b. $\displaystyle\lim_{\vartheta\to 0}\frac{\sin\vartheta}{\sqrt{\vartheta+2}-\sqrt{2}}$

c. $\displaystyle\lim_{x\to\infty}\frac{3+2x+5x^2-2x^3}{3x^2+x+7}$ d. $\displaystyle\lim_{x\to -\infty}\frac{6x+1}{\sqrt{4x^2-3}}$

2. Find a value of c such that

$$g(x)=\begin{cases}\sqrt{cx-1} & \text{if } x\leqslant 5, \text{ and}\\ 200/x^2 & \text{if } x>5,\end{cases}$$

will be continuous at 5.

3. Given the following graphs (with unit lengths marked along the coordinate axes), evaluate the following if possible. Assume that $g'(1)=1$ and $g(1)=\frac{1}{2}$.

a. $\displaystyle\lim_{x\to 0}f(x)$

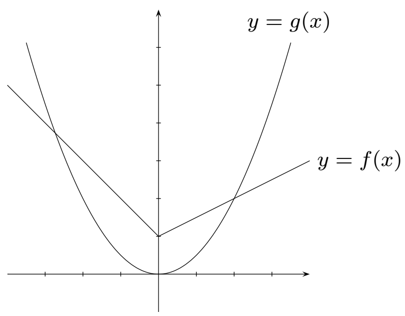
b. $\displaystyle\lim_{x\to 0^-}\frac{f(x)}{g(x)}$

c. $g'(0)$

d. $f'(0)$

e. $(f/g)'(1)$.

f. $(f\circ g)'(1)$



4. a. State the Mean Value Theorem.

b. Find all values of ξ that satisfy the conclusion of the Mean Value Theorem for $f(x)=x(\ln x)^2$ on $[\,1,e\,]$, or else explain why there are no such values.

5. Given the function

$$f(x)=\frac{x}{x+1},$$

find $f'(x)$ using the limit definition of the derivative.

6. Find dy/dx for each of the following:

a. $y=x^2+2^x+\ln|x|-\dfrac{1}{2x}+\sqrt[3]{x^2}+\sqrt{e^{2\pi}}$

b. $y=\sin(2x-3)^6-\cos^6(2x-3)$ c. $y=\left(\dfrac{3x+4}{5x^2+1}\right)^3$

d. $e^{xy}=17x-\tan y$ e. $y=(2x+3)^{2x+3}$ f. $y=\ln\dfrac{(5x+2)^2e^{5x}}{(2-\sqrt{x})^{2/3}}$

7. Determine the values of x at which the graph of g has a horizontal tangent line, where $g(x)=(x-3)^5(3x+4)^3$.

8. Find an equation for the tangent line to the graph of

$$y=\frac{3x+5}{x^2+3}$$

where $x=1$.

9. Given the curve $x^2+xy+y^2=4$.

a. Find dy/dx.

b. Determine all points (x,y) on the curve where the tangent line is parallel to the line $y=x+4$.

10. State the Product Rule for derivatives, and prove it using the limit definition of the derivative.

11. A new hydro-electric dam is built on Algonquin land in Parc de La Vérendrye. When the dam is finally closed, the flood waters spread outward in the form of a semi-circle centred at the middle of the dam, at a rate of $800{,}000\,\mathrm{m}^2$ per hour. At what rate is the radius of the flooded land increasing when $2{,}000{,}000\,\mathrm{m}^2$ of traditional native hunting grounds have been covered?

12. Sketch the graph of

$$f(x)=\frac{2+x-x^2}{(x-1)^2}.$$

13. A rectangular cage (called a battery cage) for a laying hen has a volume of $0.016\,\mathrm{m}^3$. While the European Union will have phased out battery cages by 2012 and Germany has already banned them, in Canada 98% of all hens are housed in battery cages. If the material for the base of the cage costs \$2 per m^2 and the material for the sides and the top costs \$3 per m^2, then what would be the dimensions of a lowest-cost battery cage with height $0.4\,\mathrm{m}$?

14. Find the absolute maximum and absolute minimum values of the function f on the closed interval $[\,-1,4\,]$, where $f(x)=5x^{2/3}-x^{5/3}$.

15. Consider the definite integral $\displaystyle\int_1^3(4x^2+1)\,dx$.

a. Find an approximation to the value of the integral using a Riemann sum with right endpoints and four rectangles.

b. Express the given definite integral as a limit of Riemann sums, and evaluate this limit without using the Fundamental Theorem of Calculus.

16. Find the position function $s(t)$ of a moving particle which has an acceleration function $a(t)=12t^2-3\sin t$, an initial velocity of $v(0)=0\,\mathrm{m/s}$, and an initial position of $s(0)=3\,\mathrm{m}$.

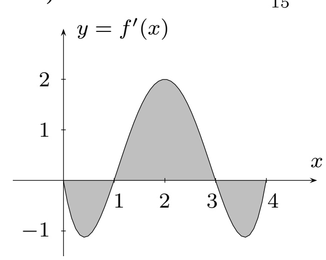
17. Evaluate the following integrals.

a. $\displaystyle\int(x^5+\sqrt[5]{x^2}-5^x+5^2)\,dx$ b. $\displaystyle\int_1^4\frac{(x+2)^2}{\sqrt{x}}\,dx$

c. $\displaystyle\int\frac{d}{dx}\sqrt{x^3+5}\,dx$ d. $\displaystyle\int\frac{3\sin^2x-2}{\sin^2x}\,dx$

18. Find the area between the graphs of $y=2+3/x$, $x=1$, $x=e$ and the x-axis.

19. The graph of f' on $[\,0,4\,]$ is shown below. The (shaded) area above the x-axis is $\frac{38}{15}$, and the total (shaded) area below the x-axis is $\frac{22}{15}$.



Given that $f(4-x)=f(x)$ and $f(0)=1$, answer the following questions.

a. Determine the all (local and global) extrema of f on $[\,0,4\,]$ (x *and* y-values).

b. How many points of inflection does f have on $[0,4]$? Justify your answer.

c. Evaluate

$$\frac{d}{dx}\left(\int_1^{\sqrt{x}}f'(t)\,dt\right)^2\Bigg|_{x=4},$$

and simplify your answer as much as possible.

1. a. Factoring and simplifying the expression in the limit gives

$$\lim_{x\to -3}\frac{x^2+x-6}{x^3+3x^2-5x-15} = \lim_{x\to -3}\frac{(x+3)(x-2)}{(x+3)(x^2-5)} = \lim_{x\to -3}\frac{x-2}{x^2-5} = -\tfrac{5}{4}.$$

b. Rationalizing the denominator gives

$$\lim_{\vartheta\to 0}\frac{\sin\vartheta}{\sqrt{\vartheta+2}-\sqrt{2}}\,\frac{\sqrt{\vartheta+2}+\sqrt{2}}{\sqrt{\vartheta}+\sqrt{2}} = \lim_{\vartheta\to 0}\left\{\frac{\sin\vartheta}{\vartheta}\left(\sqrt{\vartheta+2}+\sqrt{2}\right)\right\} = 2\sqrt{2},$$

since $\lim_{\vartheta\to 0}\frac{\sin\vartheta}{\vartheta}=1$.

c. Extracting the dominant powers of x from the numerator and denominator gives

$$\lim_{x\to\infty}\frac{3+2x+5x^2-2x^3}{3x^2+x+7} = \lim_{x\to\infty}\left\{x\cdot\frac{3/x^3+2/x^2+5/x-2}{3+1/x+7/x^2}\right\} = -\infty,$$

since the limit of the right factor is $-\frac{2}{3}$ as $x\to\infty$.

d. Extracting the dominant powers of x from the numerator and denominator gives

$$\lim_{x\to -\infty}\frac{6x+1}{\sqrt{4x^2-3}} = \lim_{x\to -\infty}\left\{\frac{x}{\sqrt{x^2}}\cdot\frac{6+1/x}{\sqrt{4-3/x^2}}\right\} = -3,$$

since the limit as $x\to -\infty$ of the right factor is 3, and $\sqrt{x^2}=-x$ if $x<0$.

2. Since

$$g(5) = \lim_{x\to 5^-} g(x) = \sqrt{5c-1}, \quad \text{and} \quad \lim_{x\to 5^+} g(x) = 8,$$

it follows that g is continuous at 5 if, and only if, $\sqrt{5c-1}=8$, or (since neither side of the equation can be negative) $5c-1=64$, which gives $5c=65$ or $c=13$.

3. a. $\lim_{x\to 0} f(x) = 1$.

b. $\lim_{x\to 0^-}\frac{f(x)}{g(x)} = \infty$, since $f(x)\to 1$ and $g(x)\to 0^+$ as $x\to 0^-$.

c. $g'(0)=0$, since the graph of g has a horizontal tangent at the origin.

d. $f'(0)$ is undefined since inspecting the graph of f reveals that

$$\frac{f(t)-f(0)}{t-0} = \begin{cases} -1 & \text{if } t<0, \text{ and} \\ \frac{1}{2} & \text{if } 0<t. \end{cases}$$

e. Inspecting the graph of f reveals that $f(1)=\frac{3}{2}$ and $f'(1)=\frac{1}{2}$. This, together with the given information $g(1)=\frac{1}{2}$ and $g'(1)=1$, and the Quotient Rule, implies that

$$\left(\frac{f}{g}\right)'(1) = \frac{f'(1)g(1)-f(1)g'(1)}{\left(g(1)\right)^2} = \frac{\frac{1}{2}\cdot\frac{1}{2}-\frac{3}{2}\cdot 1}{\left(\frac{1}{2}\right)^2} = -5.$$

f. From $f'(t)=\frac{1}{2}$ for $t>0$, $g'(1)=1$, and the Chain Rule, it follows that

$$(f\circ g)'(1) = f'(g(1))g'(1) = f'(\tfrac{1}{2})g'(1) = \tfrac{1}{2}\cdot 1 = \tfrac{1}{2}.$$

4. a. If f is continuous on $[\,a,b\,]$ and differentiable on $(\,a,b\,)$, then there is a real number ξ in $(\,a,b\,)$ such that $f(b)-f(a)=f'(\xi)(b-a)$.

b. If $f(x)=x(\ln x)^2$ then f is differentiable on $(\,0,\infty\,)$, and therefore continuous on $[\,1,e\,]$ and differentiable on $(\,1,e\,)$. So the Mean Value Theorem implies that there is a real number ξ in $(\,1,e\,)$ such that $f(e)-f(1)=f'(\xi)(e-1)$, *i.e.*,

$$e = \left((\ln\xi)^2+\ln\xi\right)(e-1), \quad \text{or} \quad (\ln\xi)^2+\ln\xi = \frac{e}{e-1}.$$

The left side of the equation is one less than $(\ln\xi+1)^2$, and hence

$$(\ln\xi+1)^2 = \frac{e}{e-1}+1 = \frac{2e-1}{e-1}, \quad \text{or} \quad \ln\xi+1 = \pm\sqrt{\frac{2e-1}{e-1}},$$

which gives $\xi = e^{-1+\sqrt{\frac{2e-1}{e-1}}}$ as the only solution in $(\,1,e\,)$.

5. If $f(x)=x/(x+1)$, then

$$\begin{aligned} f'(x) &= \lim_{t\to x}\frac{t/(t+1)-x/(x+1)}{t-x} = \lim_{t\to x}\frac{t(x+1)-x(t+1)}{(t+1)(x+1)(t-x)} \\ &= \lim_{t\to x}\frac{t-x}{(t+1)(x+1)(t-x)} = \lim_{t\to x}\frac{1}{(t+1)(x+1)} \\ &= \frac{1}{(x+1)^2}. \end{aligned}$$

6. a. Since $y = x^2+e^{x\ln 2}+\ln|x|-\frac{1}{2}x^{-1}+x^{2/3}+e^{\pi}$, it follows that

$$\frac{dy}{dx} = 2x+e^{x\ln 2}\ln 2+x^{-1}+\tfrac{1}{2}x^{-2}+\tfrac{2}{3}x^{-1/3}.$$

b. If $y=\sin(2x-3)^6-\cos^6(2x-3)$, then

$$\frac{dy}{dx} = 12(2x-3)^5\cos(2x-3)^5+12\cos^5(2x-3)\sin(2x-3).$$

c. Logarithmic differentiation gives

$$\begin{aligned} \frac{dy}{dx} &= y\frac{d}{dx}\{\log|y|\} = 3y\left\{\frac{3}{3x+4}-\frac{10x}{5x^2+1}\right\} \\ &= 3y\left\{\frac{-15x^2-40x+3}{(3x+4)(5x^2+1)}\right\} = -\frac{3(3x+4)^2(15x^2+40x-3)}{(5x^2+1)^4}. \end{aligned}$$

d. Differentiating the given equation with respect to x gives

$$e^{xy}\left(y+x\frac{dy}{dx}\right) = 17-\sec^2 y\frac{dy}{dx}, \quad \text{or} \quad \left(xe^{xy}+\sec^2 y\right)\frac{dy}{dx} = 17-ye^{xy},$$

and so

$$\frac{dy}{dx} = \frac{17-ye^{xy}}{xe^{xy}+\sec^2 y}.$$

e. If $y=(2x+3)^{2x+3}=e^{(2x+3)\log(2x+3)}$, then

$$\begin{aligned} \frac{dy}{dx} &= e^{(2x+3)\log(2x+3)}\left\{2\log(2x+3)+2(2x+3)/(2x+3)\right\} \\ &= 2(2x+3)^{2x+3}\left(1+\log(2x+3)\right). \end{aligned}$$

f. If $y=\ln\frac{(5x+2)^2e^{5x}}{(2-\sqrt{x})^{2/3}} = 2\ln|5x+2|+5x-\frac{2}{3}\ln|2-\sqrt{x}|$, then

$$\frac{dy}{dx} = \frac{10}{5x+2}+5-\frac{2}{3}\cdot\frac{-\frac{1}{2}x^{-1/2}}{2-\sqrt{x}} = \frac{10}{5x+2}+5+\frac{1}{3(2-\sqrt{x})\sqrt{x}}.$$

7. If $g(x)=(x-3)^5(3x+4)^3$, then

$$\begin{aligned} g'(x) &= 5(x-3)^4(3x+4)^3+9(x-3)^5(3x+4)^2 \\ &= (x-3)^4(3x+4)^2(24x-7), \end{aligned}$$

and so the tangent line to the graph of g is horizontal if x is $-\frac{4}{3}$, $\frac{7}{24}$ or 3.

8. If $y=\frac{3x+5}{x^2+3}$, then

$$\left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{3(x^2+3)-(3x+5)(2x)}{(x^2+3)^2}\right|_{x=1} = \left.\frac{9-10x-3x^2}{(x^2+3)^2}\right|_{x=1} = -\tfrac{1}{4},$$

and $y\big|_{x=1}=2$. Therefore, the tangent line to the graph of the given curve at the point where $x=1$ has equation $x+4y=9$.

9. a. Differentiating the given equation with respect to x yields

$$2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx}=0, \;\text{ or }\; (x+2y)\frac{dy}{dx}=-(2x+y); \;\text{ so }\; \frac{dy}{dx}=-\frac{2x+y}{x+2y}.$$

b. The line with equation $y=x+4$ has slope 1, so a line tangent to the given curve is parallel to this line if, and only if,

$$-\frac{2x+y}{x+2y}=1, \quad \textit{i.e.}, \quad 2x+y=-x-2y, \quad \text{or} \quad y=-x.$$

Replacing y by $-x$ in the given equation of the curve yields

$$x^2+x(-x)+(-x)^2=4, \quad \textit{i.e.}, \quad x^2=4, \quad \text{and so} \quad x=\pm 2 \;\text{ and }\; y=\mp 2.$$

Therefore, the line tangent to the given curve is parallel to the line with equation $y=x+4$ at the points $(2,-2)$ and $(-2,2)$.

10. If the functions f and g are differentiable at x, then so is their product fg, and $(fg)'(x)=f'(x)g(x)+f(x)g'(x)$; for

$$\begin{aligned} (fg)'(x) &= \lim_{t\to x}\frac{(fg)(t)-(fg)(x)}{t-x} \\ &= \lim_{t\to x}\frac{f(t)g(t)-f(x)g(t)+f(x)g(t)-f(x)g(x)}{t-x} \\ &= \lim_{t\to x}\left\{\frac{f(t)-f(x)}{t-x}\cdot g(t)\right\}+f(x)\cdot\lim_{t\to x}\frac{g(t)-g(x)}{t-x} \\ &= f'(x)g(x)+f(x)g'(x), \end{aligned}$$

by the linearity of limits, the product law for limits, the definitions of $f'(x)$ and $g'(x)$, and the fact that g is continuous (since it is differentiable) at x.

11. If A denotes the area (in square kilometres), and r the radius (in kilometres), of the semi-circular region of flooded land, then $A = \frac{1}{2}\pi r^2$, and so

$$\tfrac{4}{5} = \frac{dA}{dt} = \pi r \frac{dr}{dt}.$$

When $A = 2$, $r = 2/\sqrt{\pi}$, and so

$$\tfrac{4}{5} = \pi \frac{2}{\sqrt{\pi}} \frac{dr}{dt}, \quad \text{or} \quad \frac{dr}{dt} = \frac{2}{5\sqrt{\pi}}.$$

Therefore, the radius of the flooded land is increasing at a rate of $2/(5\sqrt{\pi})$ square kilometres per hour when 2 square kilometres of land have been covered.

12. The domain of f is $\mathbb{R} \setminus \{1\}$, and since $f(x) \to \infty$ as $x \to 1$, the graph of f has a vertical asymptote with equation $x = 1$. Since $f(x) \to -1$ as $x \to \pm\infty$, the graph of f has one horizontal asymptote with equation $y = -1$. The intercepts of the graph are $(0, 2)$, $(-1, 0)$ and $(2, 0)$, the latter being found by factorizing $2 + x - x^2 = (1 + x)(2 - x)$. Next, since

$$f(x) = \frac{2 + x - x^2}{(x-1)^2} = \frac{2 - (x-1) - (x-1)^2}{(x-1)^2} = \frac{2}{(x-1)^2} - \frac{1}{x-1} - 1,$$

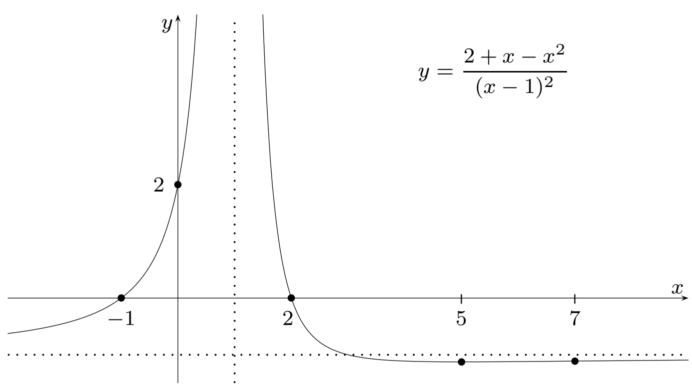
one has

$$f'(x) = -\frac{4}{(x-1)^3} + \frac{1}{(x-1)^2} = \frac{x-5}{(x-1)^3},$$

which is positive if $x < 1$ or $x > 5$, and negative if $1 < x < 5$. Therefore, f is increasing on $(-\infty, 1)$ and on $(5, \infty)$, and is decreasing on $(1, 5)$, with a local (and global) minimum at $(5, -\frac{9}{8})$. The second derivative of f is

$$f''(x) = \frac{12}{(x-1)^4} - \frac{2}{(x-1)^3} = \frac{2(7-x)}{(x-1)^4},$$

which is positive if $x < 7$ and $x \neq 1$, and negative if $x > 7$. Therefore, the graph of f is concave up on $(-\infty, 1)$ and on $(1, 7)$, and concave down on $(7, \infty)$, with a point of inflection at $(7, -\frac{10}{9})$. Below is a sketch of the graph of f, with the asymptotes drawn as dotted lines and the points of interest emphasized.



(That the graph of f meets its horizontal asymptote at the point $(3, -1)$ can be seen by solving the equation $f(x) = -1$.)

13. If ℓ and w denote, respectively, the length and width of the cage (in metres) then the volume of the gage is $\frac{2}{125} = \frac{2}{5}\ell w$, so $w = \frac{1}{25}\ell^{-1}$, and the cost of the cage is $C = 2\ell w + \frac{2}{5}\ell + \frac{2}{5}w = \frac{2}{25} + \frac{2}{5}\ell + \frac{2}{125}\ell^{-1}$, where $\ell > 0$. Then

$$\frac{dC}{d\ell} = \tfrac{2}{5} - \tfrac{2}{125}\ell^{-2} = \tfrac{2}{125}\ell^{-2}(25\ell^2 - 1), \quad \text{and} \quad \frac{d^2C}{d\ell^2} = \tfrac{4}{125}\ell^{-3}.$$

Since the only positive zero of $dC/d\ell$ is $\frac{1}{5}$, and $d^2C/d\ell^2 > 0$ if $\ell > 0$, it follows (by the Second Derivative Test for global extrema) that the smallest value of C on $(0, \infty)$ occurs at $\frac{1}{5}$. If $\ell = \frac{1}{5}$ then $w = \frac{1}{5}$, so the cheapest cage has a square base with side $\frac{1}{5}$ m and height $\frac{2}{5}$ m.

14. If $f(x) = 5x^{2/3} - x^{5/3} = x^{2/3}(5 - x)$, then

$$f'(x) = \tfrac{10}{3}x^{-1/3} - \tfrac{5}{3}x^{2/3} = \tfrac{5}{3}x^{-1/3}(2 - x),$$

which is zero if $x = 2$ and undefined if $x = 0$. Comparing $f(-1) = 6$, $f(0) = 0$, $f(2) = 3\sqrt[3]{4}$ and $f(4) = 2\sqrt[3]{2}$, reveals that the absolute minimum value of f on $[-1, 4]$ is 0 and the absolute maximum value of f on $[-1, 4]$ is 6.

15. a. Dividing $[1, 3]$ into four subintervals of equal length gives $\Delta x = \frac{1}{2}$ and $x_i = 1 + \frac{1}{2}i$, or 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3. The corresponding right endpoint sum is

$$\mathscr{R}_4 = \tfrac{1}{2}\big(f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2}) + f(3)\big) = \tfrac{1}{2}\big(10 + 17 + 26 + 37\big) = 45.$$

b. Dividing $[1, 3]$ into n subintervals of equal length gives $\Delta x = \frac{2}{n}$ and $x_i = 1 + \frac{2}{n}i$. The corresponding right endpoint sum is

$$\begin{aligned}
\mathscr{R}_n &= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left\{4\left(1 + \frac{2i}{n}\right)^2 + 1\right\} = \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left\{5 + \frac{16i}{n} + \frac{16i^2}{n^2}\right\} \\
&= \frac{10}{n}\sum_{i=1}^{n} 1 + \frac{32}{n^2}\sum_{i=1}^{n} i + \frac{32}{n^3}\sum_{i=1}^{n} i^2 \\
&= \frac{10}{n}\cdot n + \frac{32}{n^2}\cdot\frac{1}{2}n(n+1) + \frac{32}{n^3}\cdot\frac{1}{6}n(n+1)(2n+1) \\
&= 10 + 16(1 + 1/n) + \tfrac{16}{3}(1 + 1/n)(2 + 1/n),
\end{aligned}$$

and therefore,

$$\int_1^3 (4x^2 + 1)\,dx = \lim_{n\to\infty}\mathscr{R}_n = 10 + 16 + \tfrac{32}{3} = \tfrac{110}{3}.$$

16. The velocity function is

$$a = v(0) + \int_0^t (12\tau^2 - 3\sin\tau)\,d\tau = 4t^3 + 3\cos t - 3,$$

and the position function is

$$s = s(0) + \int_0^t (4\tau^3 + 3\cos\tau - 3)\,d\tau = t^4 + 3\sin t - 3t + 3.$$

17. a. Integrating term by term gives

$$\int (x^5 + \sqrt[5]{x^2} - 5^x + 5^2)\,dx = \tfrac{1}{6}x^6 + \tfrac{5}{7}\sqrt[5]{x^7} + 5^x/(\log 5) + 25x + C.$$

b. Expanding, dividing and integrating term by term, gives

$$\begin{aligned}
\int_1^4 \frac{(x+2)^2}{\sqrt{x}}\,dx &= \int_1^4 (x^{3/2} + 4x^{1/2} + 4x^{-1/2})\,dx \\
&= \left(\tfrac{2}{5}x^{5/2} + \tfrac{8}{3}x^{3/2} + 8x^{1/2}\right)\Big|_1^4 = \tfrac{586}{15}.
\end{aligned}$$

c. Since $\sqrt{x^3 + 5}$ is an antiderivative of $\dfrac{d}{dx}\sqrt{x^3 + 5}$, one has

$$\int \frac{d}{dx}\sqrt{x^3 + 5}\,dx = \sqrt{x^3 + 5} + C.$$

d. Dividing and integrating term by term gives

$$\int \frac{3\sin^2 x - 2}{\sin^2 x}\,dx = \int (3 - 2\csc^2 x)\,dx = 3x + 2\cos x + C.$$

18. Since $2 + 3/x$ is continuous and positive on $[1, e]$, the area in question is equal to

$$\int_1^e (2 + 3/x)\,dx = (2x + 3\log x)\Big|_1^e = (2e + 3) - (2 + 0) = 2e + 1.$$

19. a. Note that f has local extrema where f' changes sign—a local minimum at 1 and a local maximum at 3. Next, since $f(0) = 1$, the given areas and symmetry imply (using the second form of the Fundamental Theorem of Calculus) that

$$\begin{aligned}
f(1) &= f(0) + \int_0^1 f'(x)\,dx = 1 - \tfrac{1}{2}\cdot\tfrac{22}{15} = \tfrac{4}{15}, \\
f(3) &= f(1) + \int_1^3 f'(x)\,dx = \tfrac{4}{15} + \tfrac{38}{15} = \tfrac{14}{5}, \quad \text{and} \\
f(4) &= f(3) + \int_3^4 f'(x)\,dx = \tfrac{14}{5} - \tfrac{1}{2}\cdot\tfrac{22}{15} = \tfrac{32}{15}.
\end{aligned}$$

Therefore, the global maximum value of f on $[1, 4]$ is $\frac{14}{5}$ and the global minimum value of f on $[1, 4]$ is $\frac{4}{15}$.

b. The graph of f has three points of inflection, corresponding to the local extrema of f' (one between 0 and 1, one at 2, and one between 3 and 4).

c. Using the (first form of the) Fundamental Theorem of Calculus, one has

$$\begin{aligned}
\frac{d}{dx}\left(\int_1^{\sqrt{x}} f'(t)\,dt\right)^2 \Bigg|_{x=4} &= 2\int_1^{\sqrt{x}} f'(t)\,dt \cdot f'(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}\,\Bigg|_{x=4} \\
&= 2\int_1^2 f'(t)\,dt\cdot f'(2)\cdot\tfrac{1}{4} \\
&= 2\cdot\tfrac{1}{2}\cdot\tfrac{38}{15}\cdot 2\cdot\tfrac{1}{4} \\
&= \tfrac{19}{15}.
\end{aligned}$$