1. Evaluate each of the following limits.

a. $\displaystyle\lim_{x\to\infty} \frac{5x^3 - 7x + 12}{7x - 4x^3}$ b. $\displaystyle\lim_{x\to 2} \frac{\sqrt{x^2+5} - 3}{x-2}$

c. $\displaystyle\lim_{t\to 0} \frac{2^{t^2} - 1}{1 - \cos 3t}$ d. $\displaystyle\lim_{x\to 1^+} \left\{ \frac{1}{x-1} + \frac{1}{x^2 - 3x + 2} \right\}$

e. $\displaystyle\lim_{x\to 1} f(x)$, if $2x - 1 \leqslant f(x) \leqslant x^2$ for $0 < x < 3$.

2. The function f is defined by

$$f(x) = \begin{cases} x^2 + 3 & \text{if } x \leqslant -2, \\ -5x & \text{if } -2 < x < 5 \text{ and} \\ \dfrac{x^2}{x-6} & \text{if } x \geqslant 5 \text{ and } x \neq 6. \end{cases}$$

a. Find and classify the discontinuities of f. Justify your answers and show all relevant work.

b. At what values of x is f continuous but not differentiable? Justify your answer.

3. Given $f(x) = 3 - \dfrac{2}{x+1}$.

a. Find and simplify $\dfrac{f(2+h) - f(2)}{h}$.

b. Use the result of part a. to find $f'(2)$.

c. Use the laws of differentiation to check your answer to part b.

d. Find an equation of the normal line to the graph of f at the point where $x = 2$.

4. Find $\dfrac{dy}{dx}$ for each of the following.

a. $y = (2x-5)^4(x^2+3)^3$ b. $y = \dfrac{3}{x^4} - \sqrt[4]{x^3} + 4\log_3 x + 3^{4x} - 4e^3$

c. $x^2y^2 + x\sin y = 4\cos 3x$ d. $y = \log\left\{ \dfrac{x^2\sqrt{\sin x}}{(2x-3)^3} \right\}^4$

e. $y = (2x-1)^{\tan 3x}$

5. At which points is the tangent line to the graph of $y = \dfrac{e^{2x}}{x-3}$ horizontal?

6. Let f be a function that is differentiable on $\mathbb{R}$.

a. Give an example to show that f' need not be continuous at 0.

b. Can f' have a removable discontinuity at 0? Justify your answer completely.

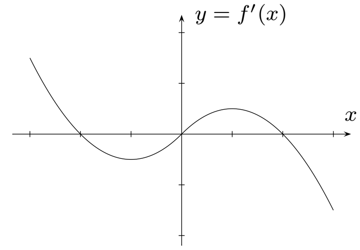
7. Find the critical numbers of $f(x) = \log(x^4 - 8x^2 + 17)$.

8. Find the absolute extrema of $f(x) = \dfrac{\sin x}{2 + \cos x}$ on the interval $[\,0, \pi\,]$.

9. Give the equations of all (vertical and horizontal) asymptotes of the graph of f, where $f(x) = \dfrac{e^{2x}}{e^{2x} - 3}$.

10. Sketch the graph of $f(x) = \frac{1}{3}x^{5/3}(8-x)$, showing clearly all (if any) asymptotes, intercepts, local extrema and points of inflection.

11. In the figure below is the graph of the derivative of a function f on the interval $[\,-3, 3\,]$, with unit lengths marked along the coordinate axes.



a. Sketch the graph of $f''(x)$.

b. Find:

i. the interval(s) on which f is increasing;

ii. the interval(s) on which the graph of f is concave down;

iii. the critical numbers of f;

iv. the x-coordinates of the point(s) of inflection of the graph of f.

12. Sand is poured into a conical pile so that the height of the pile always equal to its diameter. If the sand is poured at the constant rate of 5 cubic metres per second, at what rate is the height of the pile changing when the height is 2 metres?

13. A cylindrical package to be sent by a postal service can have a maximum combined length and girth (perimeter of its circular cross section) of 84 inches. Find the dimensions of the package with the maximum volume.

14. Determine the number of real zeros of $f(x) = x^5 + 5x^3 - 50x - 35$.

15. Evaluate each of the following integrals.

a. $\displaystyle\int_1^2 \left(x + \frac{1}{x^2} \right)^2 dx$ b. $\displaystyle\int \left(3x^7 - \sqrt{x^7} + \tfrac{1}{7}3^x - e^3 \right) dx$

c. $\displaystyle\int \frac{4\sin\vartheta}{\cos^2\vartheta}\, d\vartheta$ d. $\displaystyle\int \frac{(\sqrt{t}+3)(\sqrt{t}-3)}{3t}\, dt$

16. Evaluate $\displaystyle\frac{d}{dx}\left\{ \int_{\frac{1}{2}\pi}^{x^2} \frac{\sin t}{t}\, dt \right\} - \int_{\frac{1}{2}\pi}^{x^2} \frac{d}{dt}\left\{ \frac{\sin t}{t} \right\} dt.$

17. Given

$$f(x) = \begin{cases} |x+2| & \text{if } x \leqslant 0, \text{ and} \\ \sqrt{4 - x^2} & \text{if } 0 < x \leqslant 2. \end{cases}$$

Evaluate $\displaystyle\int_{-5}^{1} f(x)\, dx$ by interpreting it in terms of area.

18. Find the area of the region bounded by the graph of $f(x) = 1 + 2\cos x$ and the x-axis from $x = \frac{1}{6}\pi$ to $x = \pi$.

19. Given the definite integral $\displaystyle\int_2^{10} (x^2 - 3x)\, dx$.

a. Approximate the value of the integral using a partition of $[\,2, 10\,]$ into four subintervals of equal length and taking midpoints as sample points.

b. Express the integral as a limit of Riemann sums, and evaluate the limit without using the Fundamental Theorem of Calculus.

1. a. Extracting dominant powers gives

$$\lim_{x\to\infty}\frac{5x^3-7x+12}{7x-4x^3}=\lim_{x\to\infty}\frac{5-7/x^2+12/x^3}{7/x^2-4}=-\tfrac{5}{4},$$

since $1/x^p\to 0$ as $x\to\infty$ for any positive real number p.

b. Rationalizing the numerator, and simplifying the resulting expression in the limit, gives

$$\begin{aligned}\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}\cdot\frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}&=\lim_{x\to 2}\frac{x^2-4}{(x-2)(\sqrt{x^2+5}+3)}\\&=\lim_{x\to 2}\frac{x+2}{\sqrt{x^2+5}+3}\\&=\tfrac{2}{3}.\end{aligned}$$

c. Multiplying and dividing by $9\log 2(1+\cos 3t)t^2$ gives

$$\begin{aligned}&\lim_{t\to 0}\frac{2^{t^2}-1}{1-\cos 3t}\cdot\frac{9\log 2(1+\cos 3t)t^2}{9\log 2(1+\cos 3t)t^2}\\&\quad=\lim_{t\to 0}\left\{\frac{e^{t^2\log 2}-1}{t^2\log 2}\cdot\left(\frac{\sin 3t}{3t}\right)^{-2}\cdot\frac{(1+\cos 3t)\log 2}{9}\right\}\\&\quad=\tfrac{2}{9}\log 2,\end{aligned}$$

since $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ (with $x=3t$) and $\lim\limits_{x\to 0}\frac{e^x-1}{x}=1$ (with $x=t^2\log 2$).

d. Combining terms and simplifying gives

$$\begin{aligned}\lim_{x\to 1^+}\left\{\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right\}&=\lim_{x\to 1^+}\frac{x-1}{(x-1)(x-2)}\\&=\lim_{x\to 1^+}\frac{1}{x-2}\\&=-1.\end{aligned}$$

e. Since $2x-1\leqslant f(x)\leqslant x^2$ for $0<x<3$,

$$\lim_{x\to 1}(2x-1)=1\qquad\text{and}\qquad\lim_{x\to 1}x^2=1,$$

the Squeeze Theorem implies that $\lim\limits_{x\to 1}f(x)=1$.

2. a. Since the parts of f are defined by rational functions, f is continuous except possibly at -2, 5, and f is certainly discontinuous at 6, which is not in the domain of f. Since

$$f(-2)=\lim_{x\to -2^-}f(x)=(-2)^2+3=7,\ \text{and}\ \lim_{x\to -2^+}f(x)=-5(-2)=10,$$

it follows that f has a jump discontinuity at -2. Next, since

$$\lim_{x\to 5^-}f(x)=-5\cdot 5=-25\quad\text{and}\quad f(5)=\lim_{x\to 5^+}f(x)=\frac{5^2}{5-6}=-25,$$

it follows that f is continuous at 5. Finally,

$$\lim_{x\to 6^-}f(x)=-\infty\quad\text{and}\quad\lim_{x\to 6^+}f(x)=\infty$$

since $x^2\to 36$ and $x-6\to 0^{\pm}$ as $x\to 6^{\pm}$, so f has an infinite discontinuity at 6.

b. Again, since the parts of f are defined by rational functions, f is differentiable except possibly not at 5, and certainly not at -2 and 6, where f is discontinuous. Since

$$\lim_{t\to 5^-}\frac{f(t)-f(5)}{t-5}=\lim_{t\to 5}\frac{-5t+25}{t-5}=\lim_{t\to 5}(-5)=-5,$$

and

$$\begin{aligned}\lim_{t\to 5^+}\frac{f(t)-f(5)}{t-5}&=\lim_{t\to 5^+}\frac{t^2/(t-6)+25}{t-5}=\lim_{t\to 5^+}\frac{t^2+25t-150}{(t-5)(t-6)}\\&=\lim_{t\to 5^+}\frac{(t+30)(t-5)}{(t-5)(t-6)}\\&=\lim_{t\to 5^+}\frac{t+30}{t-6}\\&=-35,\end{aligned}$$

it follows that $f'(5)$ is undefined. Therefore, f is continuous but not differentiable at 5 (and is otherwise differentiable where continuous).

3. a. If $f(x)=3-2/(x+1)$, then

$$\begin{aligned}\frac{f(2+h)-f(2)}{h}&=\frac{1}{h}\left\{\left(3-\frac{2}{(2+h)+1}\right)-\left(3-\frac{2}{2+1}\right)\right\}\\&=\frac{2}{h}\left\{\frac{-1}{h+3}+\frac{1}{3}\right\}\\&=\frac{2}{3(h+3)},\quad\text{provided}\quad h\neq 0.\end{aligned}$$

b. By definition (and part a),

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\tfrac{2}{9}.$$

c. By the power rule and linearity of the derivative,

$$f'(x)=\frac{d}{dx}\left\{3-2(x+1)^{-1}\right\}=2(x+1)^{-2}=\frac{2}{(x+1)^2},$$

and therefore $f'(2)=2/(2+1)^2=\tfrac{2}{9}$.

d. The slope of the normal line to the graph of f at the point where $x=2$ is $-\tfrac{9}{2}$ (since it is perpendicular to the tangent), and the y-coordinate of the point on the graph of f where $x=2$ is $f(2)=3-2/(2+1)=\tfrac{7}{3}$. Therefore, $9x+2y=9(2)+2(\tfrac{7}{3})=\tfrac{68}{3}$, or $27x+6y=68$, is an equation of the normal line to the graph of f at the point where $x=2$.

4. a. If $y=(2x-5)^4(x^2+3)^3$, then

$$\begin{aligned}\frac{dy}{dx}&=8(2x-5)^3(x^2+3)+6x(2x-5)^4(x^2+3)^2\\&=2(2x-5)^3(x^2+3)^2(10x^2-15x+12).\end{aligned}$$

b. If

$$\begin{aligned}y&=\frac{3}{x^4}-\sqrt[4]{x^3}+4\log_3 x+3^{4x}-4e^3\\&=3x^{-4}-x^{3/4}+4(\log x)/\log 3+e^{4x\log 3}-4e^3,\end{aligned}$$

then

$$\frac{dy}{dx}=-12x^{-5}-\tfrac{3}{4}x^{-1/4}+4/(x\log 3)+e^{4x\log 3}4\log 3.$$

c. Differentiating each side of $x^2y^2+x\sin y=4\cos 3x$ with respect to x gives

$$2xy^2+2x^2y\frac{dy}{dx}+\sin y+x\cos y\frac{dy}{dx}=-12\sin 3x,$$

or

$$x(2xy+\cos y)\frac{dy}{dx}=-(12\sin 3x+\sin y+2xy^2);$$

therefore,

$$\frac{dy}{dx}=-\frac{12\sin 3x+\sin y+2xy^2}{x(2xy+\cos y)}.$$

d. If

$$y=\log\left\{\frac{x^2\sqrt{\sin x}}{(2x-3)^3}\right\}^4=8\log x+2\log(\sin x)-12\log(2x-3),$$

then

$$\frac{dy}{dx}=\frac{8}{x}+2\cot x-\frac{24}{2x-3}.$$

e. If $y=(2x-1)^{\tan 3x}=e^{(\tan 3x)\log(2x-1)}$, then

$$\begin{aligned}\frac{dy}{dx}&=e^{(\tan 3x)\log(2x-1)}\left\{3(\sec^2 3x)\log(2x-1)+\frac{2\tan 3x}{2x-1}\right\}\\&=(2x-1)^{\tan 3x-1}\left\{3(\sec^2 3x)(2x-1)\log(2x-1)+2\tan 3x\right\}.\end{aligned}$$

5. If $y=e^{2x}/(x-3)$, then

$$\frac{dy}{dx}=\frac{2e^{2x}(x-3)-e^{2x}}{(x-3)^2}=\frac{e^{2x}(2x-7)}{(x-3)^2},$$

which is zero if, and only if, $x=\tfrac{7}{2}$, where $y=2e^7$. Therefore, the tangent line to the given curve is horizontal at the point $(\tfrac{7}{2},2e^7)$.

6. a. If f is the function defined by

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0, \text{ and} \\ 0 & \text{if } x = 0, \end{cases}$$

then

$$f'(x) = \begin{cases} 2x \sin(1/x) - \cos(1/x) & \text{if } x \neq 0, \text{ and} \\ 0 & \text{if } x = 0, \end{cases}$$

which is clear except at 0, where it follows from the definition of the derivative:

$$f'(0) = \lim_{t \to 0} \frac{f(t) - f(0)}{t - 0} = \lim_{t \to 0} t \sin(1/t) = 0$$

by the Squeeze Theorem (because $-|t| \leqslant t\sin(1/t) \leqslant |t|$ for $t \neq 0$, and $\lim_{t\to 0}\big(\pm|t|\big) = 0$). So f is differentiable on $\mathbb{R}$. However, $\lim_{x\to 0} 2x\sin(1/x) = 0$ (as was seen in the calculation of $f'(0)$) and $\lim_{x\to 0}\cos(1/x)$ is undefined (since $\cos(1/x)$ oscillates between -1 and 1 as $x \to 0$), and so $\lim_{x\to 0} f'(x)$ is undefined. Therefore, f' is not continuous at 0.

b. If f is differentiable on $\mathbb{R}$ then, by the Mean Value Theorem, for every real number $x \neq 0$ there is a real number ξ_x between 0 and x such that

$$\frac{f(x) - f(0)}{x - 0} = f'(\xi_x).$$

Since ξ_x is between x and 0, $\xi_x \to 0$ as $x \to 0$, and $\xi_x \neq 0$ if $x \neq 0$. So if $\lim_{x\to 0} f'(x)$ is defined, then it follows that

$$\lim_{x\to 0} f'(x) = \lim_{x\to 0} f'(\xi_x) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = f'(0).$$

Therefore, f' cannot have a removable discontinuity at 0.

7. If $f(x) = \log(x^4 - 8x^2 + 17)$, then

$$f'(x) = \frac{4x^3 - 16x}{x^4 - 8x^2 + 17} = \frac{4x(x^2 - 4)}{x^4 - 8x^2 + 17}.$$

Since $x^4 - 8x^2 + 17 = (x^2 - 4)^2 + 1$ is never zero, f is continuous on $\mathbb{R}$. So the critical numbers of f are the zeros of f', *i.e.*, 0 and ± 2.

8. If $f(x) = (\sin x)/(2 + \cos x)$, then

$$f'(x) = \frac{(\cos x)(2 + \cos x) - (\sin x)(-\sin x)}{(2 + \cos x)^2} = \frac{1 + 2\cos x}{(2 + \cos x)^2},$$

which is defined for all real values of x and zero if $\cos x = -\frac{1}{2}$, *i.e.*, if x is $\frac{2}{3}\pi + 2\pi n$, or $\frac{4}{3}\pi + 2\pi n$, where n is an integer. So the only critical number of f in $[\,0, \pi\,]$ is $\frac{2}{3}\pi$. Comparing

$$f(0) = 0, \qquad f(\tfrac{2}{3}\pi) = \tfrac{1}{3}\sqrt{3} \qquad \text{and} \qquad f(\pi) = 0,$$

reveals that the absolute maximum value of f on $[\,0, \pi\,]$ is $f(\frac{2}{3}\pi) = \frac{1}{3}\sqrt{3}$ and the absolute minimum value of f on $[\,0, \pi\,]$ is $f(0) = f(\pi) = 0$.

9. Since $e^t \to 0$ as $t \to -\infty$, it follows that

$$\lim_{x\to -\infty} \frac{e^{2x}}{e^{2x} - 3} = 0, \quad \text{and} \quad \lim_{x\to \infty} \frac{e^{2x}}{e^{2x} - 3} = \lim_{x\to\infty} \frac{1}{1 - 3e^{-2x}} = 1,$$

so $y = 0$ and $y = 1$ are equations of the horizontal asymptotes of the graph of f. Also, since $e^{2x} \to 3^{\pm}$ as $x \to \frac{1}{2}\log 3^{\pm}$, it follows that

$$\lim_{x\to \frac{1}{2}\log 3^{\pm}} \frac{e^{2x}}{e^{2x} - 3} = \pm\infty,$$

and so $\frac{1}{2}\log 3$ is an infinite discontinuity of f. Since f is continuous at every real number except $\frac{1}{2}\log 3$, $x = \frac{1}{2}\log 3$ is the equation of the vertical asymptote of the graph of f.

10. The function f is continuous on $\mathbb{R}$, and its end behaviour is similar to that of $-\frac{1}{3}x^{8/3}$, so its graph has no asymptotes. The intercepts of the graph of f are the origin and $(8, 0)$. To compute the derivatives of f, first note that

$$f(x) = \tfrac{1}{3}x^{5/3}(8 - x) = \tfrac{8}{3}x^{5/3} - \tfrac{1}{3}x^{8/3},$$

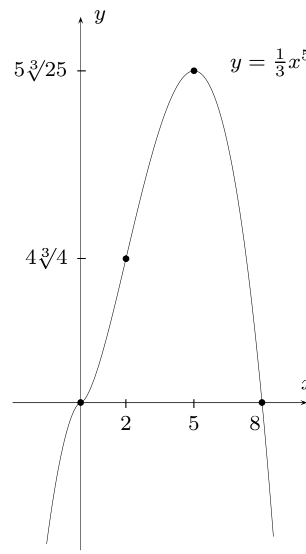
and so

$$f'(x) = \tfrac{40}{9}x^{2/3} - \tfrac{8}{9}x^{5/3} = \tfrac{8}{9}x^{2/3}(5 - x),$$

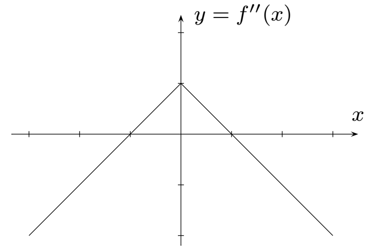
and

$$f''(x) = \tfrac{80}{27}x^{-1/3} - \tfrac{40}{27}x^{2/3} = \tfrac{40}{27}x^{-1/3}(2 - x).$$

The derivative of f is defined for all real values of x and is zero if $x = 0, 5$. Since $x^{2/3}$ is never negative, $f'(x) \geqslant 0$ if $x < 5$ and $f'(x) < 0$ if $x > 5$. Therefore, f is increasing on $(\,-\infty, 5\,)$, decreasing on $(\,5, \infty\,)$, and has a local (and global) maximum value $f(5) = \frac{1}{3}5^{5/3}(8 - 5) = 5\sqrt[3]{25}$. The second derivative of f is defined for all real values of x except 0, and is zero if $x = 2$. $f''(x)$ is negative if $x < 0$ or $x > 2$, and positive if $0 < x < 2$. Therefore, the graph of f is concave up on $(\,0, 2\,)$, concave down on $(\,-\infty, 0\,)$ and on $(\,2, \infty\,)$, and has points of inflection at the origin and at $(2, 4\sqrt[3]{4})$ (since $f(2) = \frac{1}{3}2^{5/3}(8 - x) = 4\sqrt[3]{4}$). Below is a sketch of the graph of f, with the points of interest emphasized.



11. a. Below is a sketch of the graph of f''. From the graph of f' you would not be able to tell that f' is piecewise quadratic (and so f'' is piecewise linear). What is essential is that the graph shows that f'' is positive on $(\,-1, 1\,)$, zero at ± 1, negative on $(\,-3, -1\,)$ and on $(\,1, 3\,)$, increasing on $(\,-3, 0\,)$ and decreasing on $(\,0, 3\,)$.



b. i. f is increasing on $(\,-3, -2\,)$ and on $(\,0, 2\,)$ (where f' is positive).

ii. The graph of f is concave down on $(\,-3, -1\,)$ and on $(\,1, 3\,)$ (where f'' is negative).

iii. The critical numbers of f are 0 and ± 2 (where f' is zero; note that f' is defined throughout $[\,-3, 3\,]$ so there are no critical numbers of the second kind).

iv. The graph of f has points of inflection at ± 1 (where f'' changes sign; *i.e.*, where the graph of f changes concavity).

12. If h denotes the height of the pile of sand, then the radius of the pile is $\frac{1}{2}h$. So the volume of the pile is $V = \frac{1}{3}\pi\big(\frac{1}{2}h\big)^2 h = \frac{1}{12}\pi h^3$, and therefore

$$5 = \frac{dV}{dt} = \tfrac{1}{4}\pi h^2 \frac{dh}{dt}.$$

When the $h = 2$, this gives

$$5 = \tfrac{1}{4}\pi(2)^2 \frac{dh}{dt}, \quad \text{or} \quad \frac{dh}{dt} = 5/\pi.$$

Hence, the height of the pile is increasing at a rate of $5/\pi$ metres per second when it is 2 metres high.

13. If g is the girth of the package, then its length is $84 - g$ and so its volume satisfies $4\pi V = 4\pi^2(g/2\pi)^2(84-g) = 84g^2 - g^3$, where $0 \leqslant g \leqslant 84$. Then

$$4\pi\frac{dV}{dg} = 168g - 3g^2 = 3g(56-g),$$

so the only critical number of V in $(0, 84)$ is 56. Since $V = 0$ at the endpoints, 0 and 84, of its relevant domain, it follows that the package with the maximum volume has girth 56 inches and length $84 - 56 = 28$ inches.

14. Since f is a polynomial function, it is differentiable (and therefore continuous) on $\mathbb{R}$, so the Intermediate Value Theorem and Rolle's Theorem both apply to f on any closed interval (of positive length). Observe that

$$f(-2) = -7,\ f(-1) = 9,\ f(0) = -35,\ f(2) = -63, \text{ and } f(3) = 193,$$

so the Intermediate Value Theorem implies that f has zeros in $(-2,-1)$, $(-1,0)$ and $(2,3)$. Therefore, f has at least three real zeros. On the other hand, $f'(x) = 5x^4 + 15x^2 - 50 = 5(x^2+5)(x^2-2)$, which is zero if, and only if, $x = \pm\sqrt{2}$. So f cannot have more than three zeros since Rolle's Theorem implies that between any two zeros of f there is a zero of f'. Therefore, f has exactly three real zeros.

15. a. Expanding and integrating term by term gives

$$\begin{aligned}\int_1^2 \left(x + \frac{1}{x^2}\right)^2 dx &= \int_1^2 \left(x^2 + 2x^{-1} + x^{-4}\right) dx \\ &= \left(\tfrac{1}{3}x^3 + 2\log x - \tfrac{1}{3}x^{-3}\right)\Big|_1^2 \\ &= \tfrac{21}{8} + 2\log 2.\end{aligned}$$

b. Integrating term by term yields

$$\int \left(3x^7 - \sqrt{x^7} + \tfrac{1}{7}3^x - e^3\right) dx = \tfrac{3}{8}x^8 - \tfrac{2}{9}\sqrt{x^9} + 3^x/(7\log 3) - e^3x + C.$$

c. Revising the integrand gives

$$\int \frac{4\sin\vartheta}{\cos^2\vartheta}\, d\vartheta = 4\int \sec\vartheta\tan\vartheta\, d\vartheta = 4\sec\vartheta + C.$$

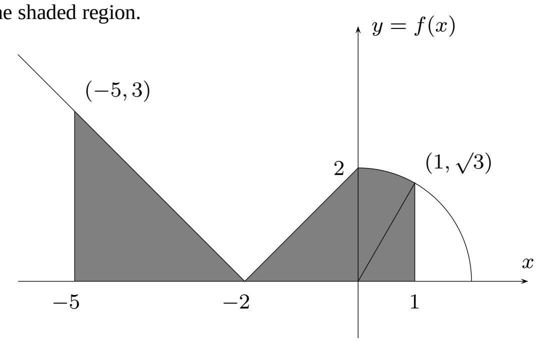
d. Expanding the numerator, dividing, and integrating term by term, gives

$$\int \frac{(\sqrt{t}+3)(\sqrt{t}-3)}{3t}\, dt = \tfrac{1}{3}\int \left(1 - 9t^{-1}\right) dt = \tfrac{1}{3}t - 3\log|t| + C.$$

16. Applying the first form of the Fundamental Theorem of Calculus (and the Chain Rule) to the first term, and the second form of the Fundamental Theorem of Calculus to the second term, gives

$$\begin{aligned}\frac{d}{dx}\left\{\int_{\frac{1}{2}\pi}^{x^2} \frac{\sin t}{t}\, dt\right\} - \int_{\frac{1}{2}\pi}^{x^2} \frac{d}{dt}\left\{\frac{\sin t}{t}\right\} dt &= \frac{\sin x^2}{x^2}\cdot\frac{d}{dx}(x^2) - \left(\frac{\sin t}{t}\right)\Big|_{\frac{1}{2}\pi}^{x^2} \\ &= \frac{\sin x^2}{x^2}\cdot 2x - \left\{\frac{\sin x^2}{x^2} - \frac{\sin\frac{1}{2}\pi}{\frac{1}{2}\pi}\right\} \\ &= \frac{\sin x^2}{x^2}(2x-1) + 2/\pi.\end{aligned}$$

17. The graph of f is sketched below, and the integral in question is equal to the area of the shaded region.



The shaded is decomposed into three triangles and a circular sector. The leftmost triangle has width 3, height 3 and area $\frac{9}{2}$; the middle triangle has height 2, width 2 and area 2; the rightmost triangle has width 1, height $\sqrt{3}$ and area $\frac{1}{2}\sqrt{3}$. The sector subtends an angle of $\frac{1}{6}\pi$ in a circle of radius 2, so its area is $\frac{1}{3}\pi$. Therefore,

$$\int_{-5}^{1} f(x)\, dx = \tfrac{9}{2} + 2 + \tfrac{1}{2}\sqrt{3} + \tfrac{1}{3}\pi = \tfrac{13}{4} + \tfrac{1}{2}\sqrt{3} + \tfrac{1}{3}\pi.$$

18. If $f(x) = 1 + 2\cos x \geqslant 0$, then $f(x) \geqslant 0$ on $[\frac{1}{6}\pi, \frac{2}{3}\pi]$ and $f(x) \leqslant 0$ on $[\frac{2}{3}\pi, \pi]$. Therefore, the area bounded by the graph of f and the x-axis on $[\frac{1}{6}\pi, \pi]$ is equal to

$$\begin{aligned}\int_{\frac{1}{6}\pi}^{\pi} |1 + 2\cos x|\, dx &= \int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} (1 + 2\cos x)\, dx - \int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} (1 + 2\cos x)\, dx \\ &= (x + 2\sin x)\Big|_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} - (x + 2\sin x)\Big|_{\frac{2}{3}\pi}^{\pi} \\ &= -1 + 2\sqrt{3} + \tfrac{1}{6}\pi.\end{aligned}$$

19. a. Dividing $[2, 10]$ into four subintervals of equal length gives $\Delta x = 2$ and $x_i = 2 + 2i$ for $i = 0, \ldots, 4$, or 2, 4, 6, 8, 10. The midpoints of the intervals in the partition are 3, 5, 7 and 9. Therefore, the midpoint sum in question is

$$\mathscr{M}_4 = 2\big\{f(3) + f(5) + f(7) + f(9)\big\} = 2\big\{0 + 10 + 28 + 54\big\} = 184.$$

b. Dividing $[2, 10]$ into n subintervals of equal length gives $\Delta x = \frac{8}{n}$, and $x_i = 2 + \frac{8}{n}i$, for $i = 0, \ldots, n$. The corresponding right endpoint sum is

$$\begin{aligned}\mathscr{R}_n &= \frac{8}{n}\sum_{i=1}^{n}\left\{\left(2 + \frac{8i}{n}\right)^2 - 3\left(2 + \frac{8i}{n}\right)\right\} \\ &= \frac{8}{n}\sum_{i=1}^{n}\left\{-2 + \frac{8i}{n} + \frac{64i^2}{n^2}\right\} \\ &= 16\left\{-\frac{1}{n}\sum_{i=1}^{n}1 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{32}{n^2}\sum_{i=1}^{n}i^2\right\} \\ &= 16\left\{-\frac{1}{n}\cdot n + \frac{4}{n^2}\cdot\frac{1}{2}n(n+1) + \frac{32}{n^3}\cdot\frac{1}{6}n(n+1)(2n+1)\right\} \\ &= 16\big\{-1 + 2(1 + 1/n) + \tfrac{16}{3}(1 + 1/n)(2 + 1/n)\big\},\end{aligned}$$

By expanding and applying the summation formulæ for powers. Therefore,

$$\int_2^{10} (x^2 - 3x)\, dx = \lim_{n\to\infty}\mathscr{R}_n = 16\big\{-1 + 2 + \tfrac{32}{3}\big\} = \tfrac{560}{3}.$$