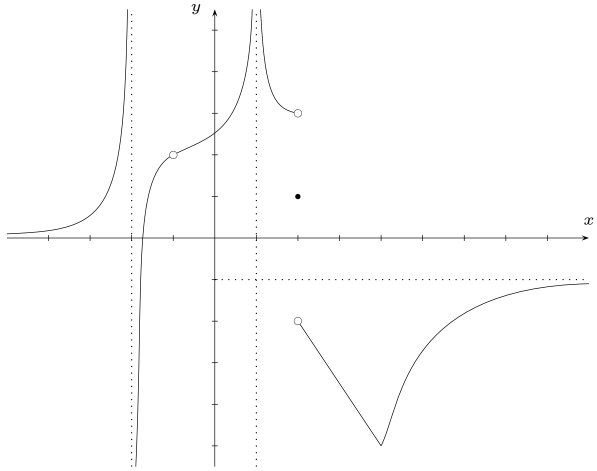
1. Refer to the following sketch (with unit lengths marked along the coordinate axes) to answer the questions below.



a. Evaluate the following. Use ∞, $-\infty$ or "does not exist" as appropriate.

i. $\lim_{x\to-2^-} f(x)$ ii. $\lim_{x\to-1} f(x)$ iii. $\lim_{x\to-\infty} f(x)$ iv. $f(2)$

v. $\lim_{x\to1} f(x)$ vi. $\lim_{x\to2} f(x)$ vii. $\lim_{x\to4} f(x)$ viii. $\lim_{x\to\infty} f(x)$

b. List the values of x at which f is discontinuous.

c. List the values of x at which f is continuous but not differentiable.

2. Evaluate the following limits. Use ∞, $-\infty$ or "does not exist" as appropriate.

a. $\lim_{x\to-2} \dfrac{x^3-4x}{3x^2+7x+2}$ b. $\lim_{x\to\infty} \dfrac{2x^3-2x^2+4x-1}{8-5x+3x^2-3x^3}$

c. $\lim_{\vartheta\to0} \dfrac{\sqrt{2\vartheta+3}-\sqrt{3}}{\sin\vartheta}$ d. $\lim_{x\to3} \dfrac{\dfrac{3}{7}-\dfrac{x}{3x-2}}{x-3}$

3. a. State the definition of the derivative of a function f.

b. Use the definition to find the derivative of $f(x) = x/(x-2)$.

c. Check your answer to Part b using the laws of differentiation.

4. Find all values of c such that

$$f(x) = \begin{cases} \dfrac{3c}{x^2-10} & \text{if } x < -4, \text{ and} \\ \sqrt{c-2x} & \text{if } -4 \leqslant x \leqslant -2. \end{cases}$$

is continuous at -4.

5. Find an equation of each line which is tangent to the graph of $y = x^2+3x+4$ and passes through the point $(2, 5)$.

6. Sketch the graph of $f(x) = x^{2/3}(x-3)$. Make sure your solution includes all intercepts, asymptotes, intervals of monotonicity and concavity, extrema, and points of inflection.

7. Find all absolute extrema of:

a. $f(x) = x^{1/3}e^{3x}$ on $[-1, 1]$; b. $g(x) = \dfrac{2\cos x}{\sin x - 2}$ on $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$.

8. Find all critical numbers of the function $y = (4x+3)^3(3x+3)^4$.

9. Find the dimensions of the rectangle of largest area which has two vertices on the x-axis and two vertices above the x-axis, bounded by the curve $y = 16-2x^2$.

10. For each of the following, find $\dfrac{dy}{dx}$.

a. $y = x^7(x^3-2x\sin 3x)^{2/3}$ b. $y = 5x^\pi - 5/x + \sqrt[5]{x^2} - \log_5 x + 5\cdot 2^x$

c. $y = \log\sqrt[3]{\tan 2x^4}$ d. $y = \sec\big(\cos(\tan \pi x)\big)$ e. $xy^2 + y\log x = x$

11. Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

a. $y = \dfrac{\sec^2(5x-2)}{x^{12}e^{-x}}$ b. $y = (\cos x)^{5x^2+1}$

12. Find an equation of the normal line to the curve $x^3y - 2y^3 - x + 3y = -11$ at the point $(-1, 2)$.

13. Let $f(x) = xe^{-2x}$. Find a simplified formula for $f^{(n)}(x)$.

14. A ladder 12 metres long is leaning against a wall. The top does not reach high enough so the bottom of the ladder gets pushed towards the wall at a rate of $10\,\text{cm/s}$. What is the rate of change of the acute angle between the ladder and the floor when the ladder reaches 8 metres up the wall?

15. Find $f(x)$ given that $f''(x) = 4-6x-4x^3$, $f(1) = 2$ and $f'(-1) = 1$.

16. For the integral $\displaystyle\int_0^{\frac{2}{3}\pi} \sin x\,dx$:

a. approximate its value by using a partition of $[0, \frac{2}{3}\pi]$ into four subintervals of equal length and taking left endpoints as sample points;

b. express it as a limit of Riemann sums, and evaluate the limit using summation formulæ (*i.e.*, without using the Fundamental Theorem of Calculus).

17. Evaluate the following integrals.

a. $\displaystyle\int_1^2 (3x+1)^2\,dx$ b. $\displaystyle\int (2^x + \sin x + \pi)\,dx$ c. $\displaystyle\int_{\frac{1}{9}\pi^2}^{\frac{1}{9}\pi^2} \sqrt{\cos\sqrt{x}}\,dx$

d. $\displaystyle\int \frac{x^3+x^2+x+1}{x}\,dx$ e. $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{\sin x}{\cos^2 x}\,dx$

18. Find the intervals of concavity of the $f(x) = \displaystyle\int_{x^2}^{0} \frac{dt}{\sqrt{1+t^2}}$.

19. Sketch the graph of

$$f(x) = \begin{cases} 2x-2 & \text{if } x \leqslant 3, \text{ and} \\ 4 & \text{if } x > 3. \end{cases}$$

and evaluate $\displaystyle\int_0^6 f(x)\,dx$ by interpreting it in terms of area.

1. a. By inspecting the given sketch: i. $\lim_{x\to -2^-} f(x) = \infty$; ii. $\lim_{x\to -1} f(x) = 2$; iii. $\lim_{x\to -\infty} f(x) = 0$; iv. $f(2) = 1$; v. $\lim_{x\to 1} f(x) = \infty$; vi. $\lim_{x\to 2} f(x)$ does not exist, because $\lim_{x\to 2^-} f(x) = 3$ and $\lim_{x\to 2^+} f(x) = -2$; vii. $\lim_{x\to 4} f(x) = -5$; viii. $\lim_{x\to\infty} f(x) = -1$. b. f has infinite discontinuities at -2 and 1, a removable discontinuity at -1, and a jump discontinuity at 2. c. f is continuous but not differentiable at 4, since $\lim_{t\to 4^-} \dfrac{f(t)-f(4)}{t-4} = -\frac{3}{2}$ and $\lim_{t\to 4^+} \dfrac{f(t)-f(4)}{t-4} > 0$.

2. a. Factoring the numerator and denominator and simplifying, gives

$$\lim_{x\to -2} \frac{x^3-4x}{3x^2+7x+2} = \lim_{x\to -2} \frac{x(x-2)(x+2)}{(3x+1)(x+2)} = \lim_{x\to -2} \frac{x(x-2)}{3x+1} = -\tfrac{8}{5}.$$

b. Extracting the dominant powers of x from the numerator and denominator gives

$$\lim_{x\to\infty} \frac{2x^3-2x^2+4x-1}{8-5x+3x^2-3x^3} = \lim_{x\to\infty} \frac{2-2/x+4/x^2-1/x^3}{8/x^3-5/x^2+3/x-3} = -\tfrac{2}{3}.$$

c. Rationalizing the numerator, and using the basic limit $\lim_{\vartheta\to 0} \dfrac{\sin\vartheta}{\vartheta} = 1$, gives

$$\lim_{\vartheta\to 0} \frac{\sqrt{2\vartheta+3}-\sqrt{3}}{\sin\vartheta} = \left(\lim_{\vartheta\to 0} \frac{\sin\vartheta}{\vartheta}\right)^{-1} \cdot \lim_{\vartheta\to 0} \frac{2}{\sqrt{2\vartheta+3}+\sqrt{3}} = \tfrac{1}{3}\sqrt{3}.$$

d. Simplifying the complex rational expression in the limit gives

$$\lim_{x\to 3} \frac{\dfrac{3}{7} - \dfrac{x}{3x-2}}{x-3} = \lim_{x\to 3} \frac{2(x-3)}{7(3x-2)(x-3)} = \lim_{x\to 3} \frac{2}{7(3x-2)} = \tfrac{2}{49}.$$

3. a. The derivative f' of a function f is defined by

$$f'(x) = \lim_{t\to x} \frac{f(t)-f(x)}{t-x},$$

and the domain of f' is the set of all real numbers x such that this limit exists.

b. By definition, the derivative of $f(x) = x/(x-2)$ is given by

$$f'(x) = \lim_{t\to x} \frac{\dfrac{t}{t-2} - \dfrac{x}{x-2}}{t-x}.$$

Simplifying this expression, and applying the independence and direct substitution properties of limits then gives

$$\begin{aligned} f'(x) &= = \lim_{t\to x} \frac{t(x-2)-x(t-2)}{(t-2)(x-2)(t-x)} = \lim_{t\to x} \frac{-2(t-x)}{(t-2)(x-2)(t-x)} \\ &= \lim_{t\to x} \frac{-2}{(t-2)(x-2)} \\ &= -\frac{2}{(x-2)^2}, \end{aligned}$$

and the domain of f' is equal to the domain of f (namely $\mathbb{R}\setminus\{2\}$).

c. After division, one obtains

$$f'(x) = \frac{d}{dx}\left\{1 + \frac{2}{x-2}\right\} = 2(-1)(x-2)^{-2} = -\frac{2}{(x-2)^2},$$

valid for $x \neq 2$ by the linearity of the derivative and the Reciprocal Rule.

4. Observe that (since we must have $c \geqslant -4$ for f to be defined on $[-4,-2]$)

$$\lim_{x\to -4^-} f(x) = \tfrac{1}{2}c \quad \text{and} \quad f(-4) = \lim_{x\to -4^+} f(x) = \sqrt{c+8}.$$

So f is continuous at -4 if, and only if, $c = 2\sqrt{c+8}$. This equation implies that $c^2 = 4(c+8)$, or $0 = c^2-4c-32 = (c-8)(c+4)$, *i.e.*, $c = 8$ or $c = -4$. However, only 8 is a solution of the original equation ($c = -4$ turns the original equation into the contradiction $-4 = 4$). Therefore, f is continuous at -4 if, and only if, $c = 8$.

5. The equation of the line tangent to the graph of $y = x^2+3x+4$ at the point where $x = \xi$ has slope $2\xi+3$, equation $y = \xi^2+3\xi+4+(2\xi+3)(x-\xi)$, and passes through the point $(2,5)$ if, and only if $5 = \xi^2+3\xi+4+(2\xi+3)(2-\xi)$, or $0 = \xi^2-4\xi-5 = (\xi-5)(\xi+1)$, *i.e.*, $\xi = 5$ or $\xi = -1$, where the slope of tangent is, respectively $2(5)+3 = 13$ and $2(-1)+3 = 1$. Therefore, the line $y = 5+13(x-2) = 13x-21$, which is tangent to the parabola at $(5,44)$, and the line $y = 5+(x-2) = x+3$, which is tangent to the parabola at $(-1,2)$, each pass through $(2,5)$, as does no other line tangent to the parabola.

6. The domain of $f(x) = x^{2/3}(x-3)$ is $\mathbb{R}$, on which f is continuous, so its graph has no vertical asymptotes. Since $f(x) = x^{5/3}(1-3/x)$ for $x \neq 0$, $f(x) \to \pm\infty$ as $x \to \pm\infty$, and the graph of f has no horizontal or oblique asymptotes, or global extrema. $f(x) = 0$ if $x = 0$ or $x = 3$, so the origin and $(3,0)$ are the intercepts of the graph of f. Since $f(x) = x^{2/3}(x-3) = x^{5/3}-3x^{2/3}$, one has

$$f'(x) = \tfrac{5}{3}x^{2/3} - 2x^{-1/3} = \tfrac{1}{3}x^{-1/3}(5x-6) \quad \text{for} \quad x \neq 0,$$

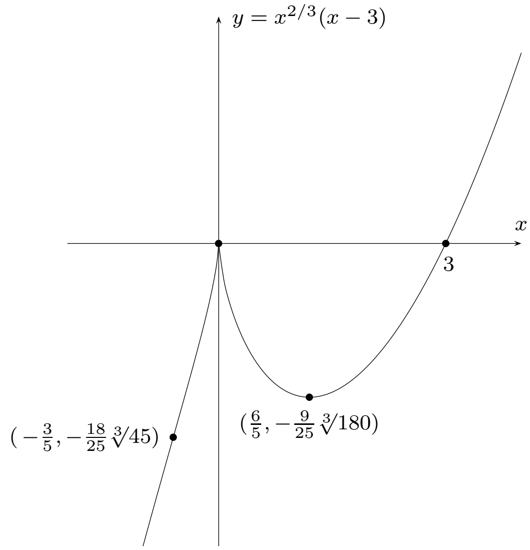
so the critical numbers of f are 0 and $\frac{6}{5}$, $f'(x) > 0$ if $x < 0$ or $x > \frac{6}{5}$ and $f'(x) < 0$ if $0 < x < \frac{6}{5}$. Therefore, f is increasing on $(-\infty, 0)$ and on $(\frac{6}{5},\infty)$, and decreasing on $(0,\frac{6}{5})$, with a local maximum at the origin and a local minimum at $(\frac{6}{5}, -\frac{9}{25}\sqrt[3]{180})$. Next,

$$f''(x) = \tfrac{10}{9}x^{-1/3} + \tfrac{2}{3}x^{-4/3} = \tfrac{2}{9}x^{-4/3}(5x+3) \quad \text{for} \quad x \neq 0,$$

so $f''(x) < 0$ if $x < -\frac{3}{5}$ and $f''(x) > 0$ if $-\frac{3}{5} < x < 0$ or $x > 0$. Therefore, f is concave down on $(-\infty, -\frac{5}{3})$, and concave up on $(-\frac{5}{3}, 0)$ and on $(0,\infty)$, with a point of inflection at $(-\frac{3}{5}, -\frac{18}{25}\sqrt[3]{45})$. Since

$$\lim_{t\to 0^\pm} \frac{f(t)-f(0)}{t-0} = \lim_{t\to 0^\pm} \frac{t^{2/3}(t-3)}{t} = \lim_{t\to 0^\pm} t^{-1/3}(t-3) = \mp\infty,$$

it follows that the graph of f has a vertical cusp at the origin. Below is a sketch of the graph of f, with the points of interest emphasized.



7. a. If $f(x) = x^{1/3}e^{3x}$, then

$$f'(x) = \tfrac{1}{3}x^{-2/3}e^{3x} + x^{1/3}e^{3x}3 = \tfrac{1}{3}x^{-2/3}e^{3x}(1+9x) \quad \text{for} \quad x \neq 0.$$

Therefore, the critical numbers of f in $[-1,1]$ are $-\frac{1}{9}$ and 0. Evaluating f at these critical numbers and at the endpoints of $[-1,1]$ gives $f(-1) = -e^{-3}$, $f(-\frac{1}{9}) = -\sqrt[3]{(9e)^{-1}}$, $f(0) = 0$, and $f(1) = e^3$. Clearly e^3 is the largest value of f on $[-1,1]$. Next, since 9 is (much) less than e^8 it follows that $e^{-9} < (9e)^{-1}$ and therefore $-\sqrt[3]{(9e)^{-1}} < -e^{-3}$, so that $-\sqrt[3]{(9e)^{-1}}$ is the smallest value of f on $[-1,1]$.

b. Since

$$\begin{aligned} g'(x) &= \frac{d}{dx}\left\{\frac{2\cos x}{\sin x - 2}\right\} = 2\cdot\frac{(-\sin x)(\sin x-2)-(\cos x)(\cos x)}{(\sin x-2)^2} \\ &= \frac{2(2\sin x-1)}{(\sin x-2)^2}, \end{aligned}$$

the only critical number of g on $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ occurs where $\sin x = \frac{1}{2}$, *i.e.*, at $\frac{1}{6}\pi$. Since $g(\pm\frac{1}{2}\pi) = 0$ and $g(\frac{1}{6}\pi) = -\frac{2}{3}\sqrt{3}$, it follows that the absolute maximum and minimum values of g on $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ are, respectively, 0 and $-\frac{2}{3}\sqrt{3}$.

8. Given $y = (4x+3)^3(3x+3)^4 = 81(4x+3)^3(x+1)^4$, one has

$$\begin{aligned} \frac{dy}{dx} &= 81\big\{12(4x+3)^2(x+1)^4 + 4(4x+3)^3(x+1)^3\big\} \\ &= 324(4x+3)^2(x+1)^3(7x+6), \end{aligned}$$

so that the critical numbers of y are -1, $-\frac{6}{7}$ and $-\frac{4}{3}$.

9. If x denotes the abscissa of the top right corner of a rectangle as described, then that rectangle has width $2x$ and height $y = 16 - 2x^2 = 2(8 - x^2)$, so its area is given by $A = 4x(8 - x^2) = 4(8x - x^3)$, for $0 \leqslant x \leqslant 2\sqrt{2}$. Since A is never negative and is zero at the endpoints of its domain (a closed interval throughout which A is continuous), and since

$$\frac{dA}{dx} = 4(8 - 3x^2),$$

it follows that the largest value of A occurs at the lone critical number, $\frac{2}{3}\sqrt{6}$, in $[\,0, 2\sqrt{2}\,]$. Therefore, the rectangle as described with the largest possible area has height $y = 2(8 - \frac{8}{3}) = \frac{32}{3}$ and width $2x = \frac{4}{3}\sqrt{6}$.

10. a. Given $y = x^7(x^3 - 2x\sin 3x)^{2/3} = x^{23/3}(x^2 - 2\sin 3x)^{2/3}$, one has

$$\begin{aligned}\frac{dy}{dx} &= \tfrac{23}{3}x^{20/3}(x^2 - 2\sin 3x) + \frac{2x^{23/3}(2x - 6\cos 3x)}{3(x^2 - 2\sin 3x)^{1/3}}\\ &= \frac{x^{20/3}(27x^2 - 12x\cos 3x - 46\sin 3x)}{3(x^2 - 2\sin 3x)^{1/3}}\end{aligned}$$

b. $\dfrac{dy}{dx} = 5\pi x^{\pi-1} + 5/x^2 + \frac{2}{5}\sqrt[5]{x^{-3}} - 1/(x\log 5) + +5(\log 2)2^x.$

c. Given $y = \log\sqrt[3]{\tan 2x^4} = \frac{1}{3}\log(\tan 2x^4)$, one has

$$\frac{dy}{dx} = \frac{8x^3\sec^2 2x^4}{3\tan 2x^4} = \frac{8x^3}{3\sin 2x^4\cos 2x^4} = \tfrac{16}{3}x^3\csc 4x^4.$$

d. $\dfrac{dy}{dx} = -\pi\sec\big(\cos(\tan\pi x)\big)\tan\big(\cos(\tan\pi x)\big)\sin(\tan\pi x)\sec^2\pi x.$

e. Differentiating $xy^2 + y\log x = x$ implicitly with respect to x gives

$$y^2 + 2xy\frac{dy}{dx} + \frac{dy}{dx}\log x + \frac{y}{x} = 1, \quad\text{or}\quad x(2xy + \log x)\frac{dy}{dx} = x(1 - y^2) - y;$$

therefore,

$$\frac{dy}{dx} = \frac{x(1 - y^2) - y}{x(2xy + \log x)}.$$

11. a. Since $\log|y| = 2\log|\sec(5x - 2)| - 12\log|x| + x$, it follows that

$$\begin{aligned}\frac{dy}{dx} &= y\frac{d}{dx}\big\{2\log|\sec(5x - 2)| - 12\log|x| + x\big\}\\ &= y\big\{10\tan(5x - 2) - 12/x + 1\big\}\\ &= x^{-13}e^x\sec^2(5x - 2)\big(10x\tan(5x - 2) + x - 12\big).\end{aligned}$$

b. Since $\log y = (5x^2 + 1)\log(\cos x)$, it follows that

$$\begin{aligned}\frac{dy}{dx} &= y\frac{d}{dx}\big\{(5x^2 + 1)\log(\cos x)\big\}\\ &= (\cos x)^{5x^2+1}\big(10x\log(\cos x) - (5x^2 + 1)\tan x\big).\end{aligned}$$

12. Differentiating the given equation implicitly with respect to y gives

$$(3x^2y - 1)\frac{dx}{dy} + x^3 - 6y^2 + 3 = 0,$$

and so slope of the line normal to the given curve at $(-1, 2)$ is equal to

$$-\frac{dx}{dy}\bigg|_{(x,y)=(-1,2)} = \frac{(-1)^3 - 6(2)^2 + 3}{3(-1)^2(2) - 1} = -\tfrac{22}{5}.$$

Therefore, an equation of the normal line in question is $22x + 5y = -12$.

13. Computing the first few derivatives of $f(x) = xe^{-2x}$ reveals a pattern.

$$\begin{aligned}f'(x) &= e^{-2x}(1 - 2x)\\ f''(x) &= e^{-2x}\big(-2 - 2(1 - 2x)\big) = (-2)e^{-2x}(2 - 2x)\\ f^{(3)}(x) &= (-2)e^{-2x}\big(-2 - 2(2 - 2x)\big) = (-2)^2e^{-2x}(3 - 2x)\end{aligned}$$

If n is a positive integer and the pattern holds for $n - 1$, then

$$\begin{aligned}f^{(n)}(x) &= \frac{d}{dx}f^{(n-1)}(x) = \frac{d}{dx}\big\{(-2)^{n-2}e^{-2x}(n - 1 - 2x)\big\}\\ &= (-2)^{n-2}e^{-2x}\big(-2 - 2(n - 1 - 2x)\big)\\ &= (-2)^{n-1}e^{-2x}(n - 2x),\end{aligned}$$

so the pattern holds for n. Therefore, by the principle of mathematical induction, $f^{(n)}(x) = (-2)^{n-1}e^{-2x}(n - 2x)$ for all $n \geqslant 0$.

14. If ϑ denotes the acute angle between the ladder and the floor, x denotes the distance (in metres) from the bottom of the ladder to the wall and y denotes the distance (in metres) from the top of the ladder to the floor, then $x = 12\cos\vartheta$ and $y = 12\sin\vartheta$. Differentiating the first relation with respect to time gives

$$-\frac{1}{10} = \frac{dx}{dt} = -12\sin\vartheta\,\frac{d\vartheta}{dt} = -y\frac{d\vartheta}{dt}, \qquad\text{or}\qquad \frac{d\vartheta}{dt} = \frac{1}{10y}.$$

Therefore, when the top of the ladder reaches 8 metres up the wall, the acute angle it makes with the floor is increasing at a rate of $1/80$ radians per second.

15. We have

$$\begin{aligned}f'(x) &= f'(-1) + \int_{-1}^x f''(t)\,dt = 1 + \int_{-1}^x (4 - 6t - 4t^3)\,dt\\ &= 1 + \big\{4t - 3t^2 - t^4\big\}\Big|_{-1}^x = 9 + 4x - 3x^2 - x^4, \quad\text{and}\\ f(x) &= f(1) + \int_1^x f'(t)\,dt = 2 + \int_1^x (9 + 4t - 3t^2 - t^4)\,dt\\ &= 2 + \big\{9t + 2t^2 - t^3 - \tfrac{1}{5}t^5\big\}\Big|_1^x = -\tfrac{39}{5} + 9x + 2x^2 - x^3 - \tfrac{1}{5}x^5.\end{aligned}$$

16. a. Dividing $[\,0, \frac{2}{3}\pi\,]$ into four subintervals of equal length gives $\Delta x = \frac{1}{6}\pi$, $x_i = \frac{1}{6}\pi i$, for $i = 0, \ldots, 4$, *i.e.*, $0, \frac{1}{6}\pi, \frac{1}{3}\pi, \frac{1}{2}\pi, \frac{2}{3}\pi$, and so the corresponding left endpoint sum is

$$\begin{aligned}\mathscr{L}_4 &= \tfrac{1}{6}\pi\big\{\sin 0 + \sin\tfrac{1}{6}\pi + \sin\tfrac{1}{3}\pi + \sin\tfrac{1}{2}\pi\big\}\\ &= \tfrac{1}{6}\pi\big\{0 + \tfrac{1}{2} + \tfrac{1}{2}\sqrt{3} + 1\big\}\\ &= \tfrac{1}{4}\pi + \tfrac{1}{12}\pi\sqrt{3}.\end{aligned}$$

b. The integral is a limit of regular right endpoint sums:

$$\int_0^{\frac{2}{3}\pi}\sin x\,dx = \lim_{n\to\infty}\tfrac{2\pi}{3n}\sum_{i=1}^n \sin\tfrac{2\pi i}{3n}.$$

Applying the identity $2\sin\alpha\sin\beta = \cos(\beta - \alpha) - \cos(\beta + \alpha)$, to the terms of the second sum below, and then to the simplified sum, one finds that

$$\begin{aligned}2\sin\tfrac{1}{2}\vartheta\sum_{i=1}^n\sin i\vartheta &= \sum_{i=1}^n 2\sin\tfrac{1}{2}\vartheta\sin i\vartheta = \sum_{i=1}^n\big\{\cos(i - \tfrac{1}{2})\vartheta - \cos(i + \tfrac{1}{2})\vartheta\big\}\\ &= \cos\tfrac{1}{2}\vartheta - \cos(n + \tfrac{1}{2})\vartheta = 2\sin\tfrac{1}{2}n\vartheta\,\sin\tfrac{1}{2}(n + 1)\vartheta,\end{aligned}$$

and therefore, with $\vartheta = \frac{2\pi}{3n}$, that

$$\begin{aligned}\int_0^{\frac{2}{3}\pi}\sin x\,dx &= \lim_{n\to\infty}\tfrac{2\pi}{3n}\sum_{i=1}^n\sin\tfrac{2\pi i}{3n} = 2\lim_{n\to\infty}\frac{\sin\frac{1}{3}\pi\,\sin\frac{1}{3}(1 + \frac{1}{n})\pi}{\frac{\sin\frac{\pi}{3n}}{\frac{\pi}{3n}}}\\ &= 2\sin^2\tfrac{1}{3}\pi = \tfrac{3}{2}.\end{aligned}$$

17. a. Expanding and integrating term by term gives

$$\int_1^2 (3x + 1)^2\,dx = \int_1^2 (9x^2 + 6x + 1)\,dx = (3x^3 + 3x^2 + x)\Big|_1^2 = 31.$$

b. Integrating term by term gives

$$\int (2^x + \sin x + \pi)\,dx = 2^x/(\log 2) - \cos x + \pi x + C.$$

c. Since the integral over $[\,a, a\,]$ a function defined at a is zero,

$$\int_{\frac{1}{9}\pi^2}^{\frac{1}{9}\pi^2}\sqrt{\cos\sqrt{x}}\,dx = 0.$$

d. Dividing and integrating term by term gives

$$\begin{aligned}\int\frac{x^3 + x^2 + x + 1}{x}\,dx &= \int (x^2 + x + 1 + 1/x)\,dx\\ &= \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + x + \log|x| + C.\end{aligned}$$

e. Revising the integrand and using a standard integral formula gives

$$\int_0^{\frac{1}{3}\pi}\frac{\sin x}{\cos^2 x}\,dx = \int_0^{\frac{1}{3}\pi}\sec x\tan x\,dx = \sec x\,\Big|_0^{\frac{1}{3}\pi} = 1.$$

18. We have

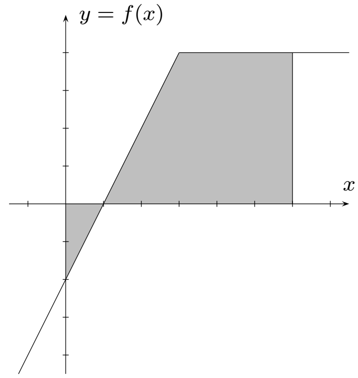
$$f'(x) = -\frac{d}{dx}\int_0^{x^2} \frac{dt}{\sqrt{1+t^2}} = -\frac{2x}{\sqrt{1+x^4}},$$

by the Chain Rule and the (First) Fundamental Theorem of Calculus, and

$$f''(x) = -2\left\{\frac{1}{\sqrt{1+x^4}} - \frac{2x^4}{\sqrt{(1+x^4)^3}}\right\} = \frac{2(x^4-1)}{\sqrt{(1+x^4)^3}},$$

so $f''(x)$ is positive if $|x| > 1$ and negative if $|x| < 1$. Therefore, the graph of f is concave up on $(-\infty, -1)$ and on $(1, \infty)$, and concave down on $(-1, 1)$.

19. Here is a sketch of the graph of f, with the region representing the integral shaded.



The region below the x-axis is a triangle with base 1, height 2, and area is 1. The region above the x-axis decomposes naturally into a triangle with base 2, height 4, and area 4, and a rectangle with base 3, height 4, and area 12. Therefore,

$$\int_0^6 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^3 f(x)\,dx + \int_3^6 f(x)\,dx = -1+4+12 = 15.$$