1. Evaluate each of the following limits.

a. $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right)$ b. $\lim_{x\to-\infty}\frac{\sqrt{x^2-3}}{4x+1}$ c. $\lim_{x\to 0}\frac{\frac{1}{x+2}-\frac{2}{x+4}}{\sin x}$

d. $\lim_{x\to 3}\frac{2-\sqrt{7-x}}{x^2-5x+6}$ e. $\lim_{t\to 0}\frac{\log(e+t)-1}{t}$

2. Find all asymptotes of the curve defined by $y=\frac{3e^x+1}{5e^x-2}$.

3. Find all values of a and b, if any, so that the function defined by

$$f(x)=\begin{cases}x^2+6x+12 & \text{if } x<-2, \text{ and}\\ ax+b & \text{if } x\geqslant -2,\end{cases}$$

is differentiable everywhere.

4. Determine a ratio $f(x)$ of quadratic polynomials such that

$$\lim_{x\to\infty} f(x)=1, \qquad \lim_{x\to 2^-} f(x)=\infty$$

and $\lim_{x\to 3} f(x)$ is defined but $f(3)$ is undefined.

5. Find $\frac{dy}{dx}$ for each of the following.

a. $y=\frac{x^9}{9}-\frac{9}{x^9}+9^x+\log_9(x)+\sqrt[9]{x}+9^9$ b. $y=\frac{(\log x)(\sec(2x-1))}{7}$

c. $y=\sqrt{(e^x-3)^2+\frac{x}{\sin x}}$ d. $y=\tan(x^9+x^x)$

6. If $g'(3)=2$ and the slope of the line tangent to the graph of $y=xg(x)$ at the point where $x=3$ is 14, determine the value of $g(3)$.

7. Allan and Balla both leave the same spot at the same time. Allan walks east at a rate of $\frac{3}{2}$ km/h, while Balla jogs $\frac{1}{3}\pi$ radians north of east at a rate of 4 km/h. How fast is the distance between them increasing after two hours?

8. Find an equation of the line tangent to the curve defined by

$$x^2+y^2=(2x^2+2y^2-1)^2$$

at the point $\left(0,\frac{1}{2}\right)$.

9. a. State the mean value theorem.

b. Let f be an everywhere twice-differentiable function with at least two critical numbers. Prove that f'' has at a root.

10. Find the intervals of monotonicity and all local extreme values of

$$y=9x^{2/3}(x-20).$$

11. Find the intervals of concavity and all points of inflection of the graph of f, where $f(x)=(1+\sin x)^{2/3}$.

12. If possible, sketch the graph of a function f with domain $\mathbb{R}\setminus\{0\}$ and the following properties. The derivative $f'(x)$ is positive if $x<-5$ or $x>6$, and negative if $-5<x<0$ or $0<x<6$. The second derivative $f''(x)$ is positive if $x<-5$, $-5<x<-3$ or $0<x<10$, and negative if $-3<x<0$ or $x>10$. Also, $f(-1)=f(2)=f(8)=0$, $f(-5)=7$, $f(-3)=5$, $f(6)=-3$, $f(10)=3$, $\lim_{x\to-\infty} f(x)=1$, $\lim_{x\to 0^-} f(x)=-\infty$ and $\lim_{x\to\infty} f(x)=6$.

Note. — The foregoing question was on the exam. On the department web page, the question on the exam is replaced by one in which the x intercepts of the curve are -1, 2 and 9, and the value of f at 10 is changed to $f(10)=4$. As the two versions have (importantly) different solutions, each is included.

13. A 600 m by 800 m rectangular park has a straight bicycle path cutting through it, one side of which is a diagonal of the park; next to this side of the path will be a rectangular playground in the park, with sides parallel to those of the park. What dimensions of the playground will maximize its area?

14. Verify the integral formula

$$\int\frac{dx}{(x^2+4)^{3/2}}=\frac{x}{4\sqrt{x^2+4}}+C.$$

15. Evaluate the integral

$$\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx$$

by interpreting it in terms of area.

16. Express the integral

$$\int_0^{\pi} x\cos(2x)\,dx$$

as a limit of Riemann sums. For ten bonus marks, evaluate the integral without using the (second) fundamental theorem of calculus.

17. Express

$$\lim_{n\to\infty}\left\{\frac{4}{n}\sum_{i=1}^{n}\left(5+\frac{4}{n}i\right)\sin\left(2+\frac{4}{n}i\right)\right\}$$

as a definite integral with lower limit 3.

18. Evaluate each of the following integrals.

a. $\int(\sqrt{x}-3)^2\,dx$ b. $\int\frac{\tan\vartheta-\cos^2\vartheta+\sec\vartheta}{\cos\vartheta}\,d\vartheta$

c. $\int_1^2\left(e^y-\frac{3}{y^2}\right)dy$ d. $\int_5^5\cot(3x^2-9)\,dx$

19. Find $\frac{dy}{dx}$ if

$$\int_0^y e^{-t}\,dt=4+\int_2^{x^2}\sin^2(t)\,dt.$$

20. Mark each statement as true or false. Justify your answers.

– If $\lim_{x\to a} f(x)=\lim_{x\to a} g(x)$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=1$.

– If f is differentiable at a, then $\lim_{x\to a} f(x)$ exists.

– If f is increasing on $[a,b]$ then $f'(x)>0$ for $a<x<b$.

– If f and g are continuous on $[a,b]$, then

$$\int_a^b f(x)g(x)\,dx=\int_a^b f(x)\,dx\int_a^b g(x)\,dx.$$

– If $f'(x)>0$ wherever $f(x)$ is defined, then f is increasing on its domain.

1. a. Since
$$\frac{1}{x}-\frac{1}{\sqrt{x}}=\left(\frac{1}{\sqrt{x}}-1\right)\frac{1}{\sqrt{x}}, \qquad \text{and} \qquad \lim_{x\to 0^+}\frac{1}{\sqrt{x}}=\infty,$$
it follows that
$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right)=\infty.$$
b. Since (recall that $\sqrt{x^2}=-x$ if $x<0$)
$$\frac{\sqrt{x^2-3}}{4x+1}=-\frac{\sqrt{1-3/x^2}}{4+1/x}, \qquad \text{and} \qquad \lim_{x\to-\infty}\frac{1}{x^n}=0,$$
for $n=1,2,3,\ldots$, it follows that
$$\lim_{x\to-\infty}\frac{\sqrt{x^2-3}}{4x+1}=-\tfrac{1}{4}.$$
c. Since
$$\frac{1}{x+2}-\frac{2}{x+4}=\frac{-x}{(x+2)(x+4)}, \qquad \text{and} \qquad \lim_{x\to 0}\frac{x}{\sin x}=1,$$
it follows that
$$\lim_{x\to 0}\frac{\dfrac{1}{x+2}-\dfrac{2}{x+4}}{\sin x}=-\tfrac{1}{8}.$$
d. Since
$$\frac{2-\sqrt{7-x}}{x^2-5x+6}=\frac{x-3}{2+\sqrt{7-x}}\cdot\frac{1}{(x-3)(x-2)},$$
it follows that
$$\lim_{x\to 3}\frac{2-\sqrt{7-x}}{x^2-5x+6}=\tfrac{1}{4}.$$
e. By the definition of the derivative,
$$\lim_{t\to 0}\frac{\log(e+t)-1}{t}=\frac{d}{dx}(\log x)\bigg|_{x=e}=\frac{1}{e}.$$

2. Since $e^t\to 0$ as $t\to-\infty$, and
$$y=\frac{3e^x+1}{5e^x-2}=\frac{3+e^{-x}}{5-2e^{-x}},$$
it follows that $y\to-\frac{1}{2}$ as $x\to-\infty$ and $y\to\frac{3}{5}$ as $x\to\infty$. Moreover, y is a continuous function of x except at $\ln\left(\frac{2}{5}\right)$, where $3e^x+1=\frac{11}{5}>0$ and $5e^x-2\to 0^\pm$; so $y\to\pm\infty$, as $x\to\ln\left(\frac{2}{5}\right)^\pm$. Thus, the horizontal asymptotes of of the curve are defined by $y=-\frac{1}{2}$ and $y=\frac{5}{3}$, and the vertical asymptote of the curve is defined by $x=\ln\left(\frac{2}{5}\right)$.

3. Let $g(x)=ax+b$ and $h(x)=x^2+6x+12$. Since g and h are polynomial functions, f is differentiable everywhere if, and only if, $g(-2)=h(-2)$ and $g'(-2)=h'(-2)$. Equivalently, $-2a+b=4$ and $a=2$, which then gives $b=8$.

4. As f has a removable discontinuity at 3, the numerator and denominator of $f(x)$ are divisible by $x-3$, and since f has an infinite discontinuity at 2, the denominator of $f(x)$ is divisible by $x-2$. Also, since $f(x)\to 1$ as $x\to\infty$, the numerator and denominator of $f(x)$ have the same leading coefficients, which may be taken to be 1, as in
$$f(x)=\frac{(x-\alpha)(x-3)}{(x-2)(x-3)},$$
where $\alpha>2$ to insure that $f(x)\to\infty$ as $x\to 2^-$. Of such expressions, the one in which $\alpha=3$ is especially simple.

5. a. If $y=\dfrac{x^9}{9}-\dfrac{9}{x^9}+9^x+\log_9(x)+\sqrt[9]{x}+9^9$, then
$$\frac{dy}{dx}=x^8+81x^{-10}+9^x\log 9+\tfrac{1}{9}\sqrt[9]{x}^{-8}.$$
b. If $y=\frac{1}{7}(\log x)(\sec(2x-1))$, then
$$\frac{dy}{dx}=\tfrac{1}{7}\left(x^{-1}\sec(2x-1)+2(\log x)\sec(2x-1)\tan(2x-1)\right).$$
c. If $y=\sqrt{(e^x-3)^2+x/(\sin x)}$, then
$$\frac{dy}{dx}=\frac{2e^x(e^x-3)+1/(\sin x)-x(\cos x)/(\sin x)^2}{2\sqrt{(e^x-3)^2+x/(\sin x)}}.$$
d. If $y=\tan(x^9+x^x)$, then
$$\frac{dy}{dx}=(9x^8+x^x(\log x+1))\sec^2(x^9+x^x),$$
since $\frac{d}{dx}(x^x)=x^x\,\frac{d}{dx}(x\log x)=x^x(\log x+1)$.

6. Since $g'(3)=2$, and
$$14=\frac{d}{dx}\big(xg(x)\big)\bigg|_{x=3}=g(3)+3g'(3)=g(3)+6,$$
it follows that $g(3)=8$.

7. By the law of cosines, the distance between the pair after t hours is
$$\sqrt{\left(\tfrac{3}{2}\right)^2+4^2-2\cdot\tfrac{3}{2}\cdot 4\cos\left(\tfrac{1}{3}\pi\right)}\cdot t=\tfrac{7}{2}t,$$
which is increasing at a rate of $\frac{7}{2}$km/hr after two hours (or any time $t>0$).

8. The given equation is equivalent to
$$0=(2r^2-1)^2-r^2=(2r+1)(r-1)(2r-1)(r+1),$$
where r is the distance between (x,y) and the origin, so the curve is the union of two circles, of radii $\frac{1}{2}$ and 1, each centred at the origin. Therefore, the tangent line at the point $\left(0,\frac{1}{2}\right)$ is horizontal, with equation $2y=1$.

9. a. If $a<b$, the function f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there is a real number ξ such that $a<\xi<b$ and $f(b)-f(a)=f'(\xi)(b-a)$.
b. As f is twice-differentiable and has two critical numbers, there are real numbers $a<b$ such that $f'(a)=f'(b)=0$. The mean value theorem, applied to f' on $[a,b]$, yields a real number ξ such that $a<\xi<b$ and $0=f''(\xi)(b-a)$, or equivalently, $f''(\xi)=0$, since $a\neq b$. Therefore, f'' has a root.

10. If $y=9x^{2/3}(x-20)=9(x^{5/3}-20x^{2/3})$ then
$$\frac{dy}{dx}=9\left(\tfrac{5}{3}x^{2/3}-\tfrac{40}{3}x^{-1/3}\right)=15x^{-1/3}(x-8),$$
which is zero if x is 8, undefined if x is zero, positive if $x<0$ or $x>8$ and negative if $0<x<8$. Therefore, y is increasing on $(-\infty,0]$ and on $[8,\infty)$ (**not on the union** $(-\infty,0]\cup[8,\infty)$ since, for example $-1<9$ but $f(-1)=-189>-297\sqrt[3]{3}=f(9)$), and y is decreasing on $[0,8]$, with a local maximum at the origin and a local minimum at $(8,-432)$.

11. If $f(x)=(1+\sin x)^{2/3}$ then
$$f'(x)=\frac{2\cos x}{3(1+\sin x)^{1/3}},$$
provided $x\neq\frac{3}{2}\pi+2k\pi$ for any integer k; moreover,
$$\begin{aligned}f'\left(\tfrac{3}{2}\pi\right)&=\lim_{x\to\frac{3}{2}\pi}\frac{(1+\sin x)^{2/3}}{x-\frac{3}{2}\pi}=\lim_{t\to 0}\frac{(1-\cos t)^{2/3}}{t}\\&=\lim_{t\to 0}\frac{\left(2\sin^2(t/2)\right)^{2/3}}{t}=\frac{1}{2^{2/3}}\lim_{t\to 0}\left\{t^{1/3}\left(\frac{\sin(t/2)}{t/2}\right)^{4/3}\right\}\\&=0,\end{aligned}$$
where $t=x-\frac{3}{2}\pi$, and thus $f'\left(\frac{3}{2}\pi+2k\pi\right)=0$ for any integer k. Next,
$$\begin{aligned}f''(x)&=\tfrac{2}{3}\left(-(\sin x)(1+\sin x)^{-1/3}-\tfrac{1}{3}(\cos x)^2(1+\sin x)^{-4/3}\right)\\&=-\frac{2}{9}\cdot\frac{3\sin^2 x+3\sin x+\cos^2 x}{(1+\sin x)^{4/3}}=-\frac{2}{9}\cdot\frac{2\sin^2 x+3\sin x+1}{(1+\sin x)^{4/3}}\\&=-\frac{2(2\sin x+1)}{9(1+\sin x)^{1/3}},\end{aligned}$$

provided $x \neq \frac{3}{2}\pi + 2k\pi$ for any integer k. Moreover,

$$f''\left(\tfrac{3}{2}\pi\right) = \lim_{x\to\frac{3}{2}\pi}\left\{\frac{2}{3(1+\sin x)^{1/3}}\cdot\frac{\cos x}{x-\frac{3}{2}\pi}\right\}$$

is undefined, since

$$\lim_{x\to\frac{3}{2}\pi}(1+\sin x)^{1/3} = 0 \quad\text{and}\quad \lim_{x\to\frac{3}{2}\pi}\frac{\cos x}{x-\frac{3}{2}\pi} = \lim_{t\to 0}\frac{\sin t}{t} = 1,$$

where $t = x - \frac{3}{2}\pi$, and thus $f''\left(\frac{3}{2}\pi + 2k\pi\right)$ is undefined for any integer k. Since $1+\sin x$ is never negative, where $f''(x)$ is defined and non-zero its sign is opposite that of $2\sin x + 1$. So $f''(x) > 0$ if $-1 < \sin x < -\frac{1}{2}$, *i.e.*, where

$$\tfrac{7}{6}\pi < x - 2k\pi < \tfrac{3}{2}\pi \qquad\text{or}\qquad \tfrac{3}{2}\pi < x - 2k\pi < \tfrac{11}{6}\pi,$$

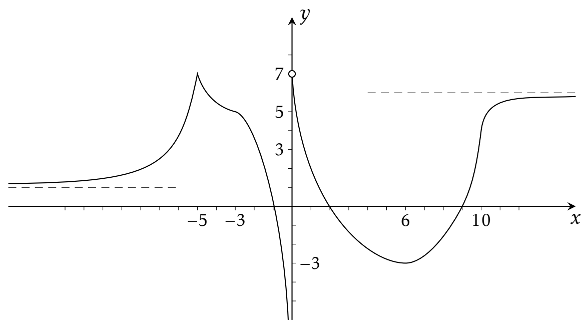
for some integer k, and $f''(x) < 0$ if $\sin x > -\frac{1}{2}$, *i.e.*, where

$$-\tfrac{1}{6}\pi < x - 2k\pi < \tfrac{7}{6}\pi,$$

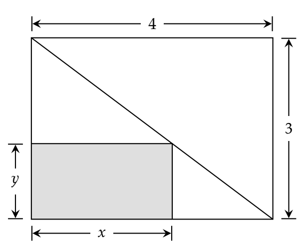
for some integer k; otherwise, $f''(x)$ is zero or undefined. It follows that the graph of f is concave up on the intervals $\left[\frac{7}{6}\pi + 2k\pi, \frac{11}{6}\pi + 2k\pi\right]$, where k is an integer, and concave down on the intervals $\left[-\frac{1}{6}\pi + 2k\pi, \frac{7}{6}\pi + 2k\pi\right]$, where k is an integer. The curve has points of inflection at $\left(\frac{7}{6}\pi + 2k\pi, \sqrt[3]{\frac{1}{4}}\right)$ and $\left(\frac{11}{6}\pi + 2k\pi, \sqrt[3]{\frac{1}{4}}\right)$, where k is an integer.

12. It is not possible to sketch the graph because there is no such function. For any such function, the mean value theorem implies that there is a real number α such that $6 < \alpha < 8$ and $3 = 2f'(\alpha)$, a real number β such that $8 < \beta < 10$ and $3 = 2f'(\beta)$, and a real number γ such that $\alpha < \gamma < \beta$, and $0 = f'(\beta) - f'(\alpha) = f''(\gamma)(\beta - \alpha) > 0$, which is plainly absurd.

In the "Trumped up" version of the exercise, which was not on the exam, the largest x intercept is changed to 9 and $f(10)$ is changed to 4. A sketch of part of the graph of one such function is displayed below.



13. In the figure below, the outer rectangle represents the park and the shaded rectangle represents the playground, in units of 200 metres.



By similarity, $4y = 3(4-x)$, and the area of the playground is

$$xy = \tfrac{3}{4}x(4-x) = 3 - \tfrac{3}{4}(x-2)^2,$$

which is maximized if $x = 2$, or 400 metres, and $y = \frac{3}{2}$, or 300 metres (the largest area being 3, or 120,000 square metres).

Alternatively, observe that the area of the playground is a quadratic function of each side which is zero at the extremes and otherwise positive, so it is maximized if each side is half the corresponding side of the park.

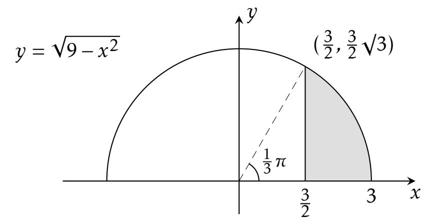
14. Since

$$\frac{d}{dx}\left\{\frac{x}{4\sqrt{x^2+4}}\right\} = \frac{1}{4\sqrt{x^2+4}} - \frac{x^2}{4(x^2+4)^{3/2}} = \frac{1}{(x^2+4)^{3/2}},$$

it follows that

$$\int\frac{dx}{(x^2+4)^{3/2}} = \frac{x}{4\sqrt{x^2+4}} + C.$$

15. The integral is the area of the region shaded in the figure below.



The region is the difference of a circular sector of radius 3 and angle $\frac{1}{3}\pi$, and a triangle of base $\frac{3}{2}$ and height $\frac{3}{2}\sqrt{3}$. The area of the sector is $\frac{1}{2}\cdot 3^2\,\frac{1}{3}\pi = \frac{3}{2}\pi$ and the area of the triangle is $\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}\sqrt{3} = \frac{9}{8}\sqrt{3}$. Therefore,

$$\int_{\frac{3}{2}}^{\frac{3}{2}\sqrt{3}}\sqrt{9-x^2}\,dx = \tfrac{3}{2}\pi - \tfrac{9}{8}\sqrt{3}.$$

16. If the interval $[0,\pi]$ is divided into k subintervals of equal length, then the length of each subinterval is π/k and the endpoints of the subintervals are $\pi j/k$ for $j = 0,1,2,\ldots,k$. The Riemann sums obtained by evaluating the integrand at the right endpoint of each subinterval are

$$\mathscr{R}_k = \frac{\pi}{k}\sum_{j=1}^{k}\frac{\pi j}{k}\cos\left(\frac{2\pi j}{k}\right), \qquad\text{and}\qquad \int_0^{\pi}x\cos(2x)\,dx = \lim_{k\to\infty}\mathscr{R}_k.$$

To evaluate the limit, first note that the sum and difference identities of the sine function give

$$2\sin\left(\tfrac{1}{2}\beta\right)\cos(\beta i) = \sin\left((i+\tfrac{1}{2})\beta\right) - \sin\left((i-\tfrac{1}{2})\beta\right),$$

and then summing over $i = j, j+1, j+2, \ldots, k$ gives

$$2\sin\left(\tfrac{1}{2}\beta\right)\sum_{i=j}^{k}\cos(\beta i) = \sin\left((k+\tfrac{1}{2})\beta\right) - \sin\left((j-\tfrac{1}{2})\beta\right).$$

Let $\beta = 2\pi/k$ and sum over $j = 1,2,3,\ldots,k$; then $2\sin(\pi/k)\mathscr{R}_k$ is π/k^2 times

$$k\sin\left((k+\tfrac{1}{2})2\pi/k\right) - \sum_{j=1}^{k}\sin\left((j-\tfrac{1}{2})2\pi/k\right),$$

or

$$k\sin(\pi/k) - \sum_{j=1}^{k}\sin(2\pi j/k - \pi/k),$$

and by symmetry (cancelling terms corresponding to j and to $k+1-j$) the second sum is equal to zero. So if $k = 2,3,4,\ldots$, then

$$\mathscr{R}_k = \frac{\pi^2}{k^2}\cdot\frac{k\sin(\pi/k)}{2\sin(\pi/k)} = \frac{\pi^2}{2k}, \quad\text{and hence}\quad \int_0^{\pi}x\cos(2x)\,dx = \lim_{k\to\infty}\mathscr{R}_k = 0.$$

17. If the interval $[3,7]$ is divided into n subintervals of equal length, then the length of each subinterval is $4/n$ and the endpoints of the subintervals are $3+4i/n$. In addition, $5+4i/n = (3+4i/n)+2$ and $2+4i/n = (3+4i/n)-1$, for $i = 0,1,2,\ldots,n$. Hence, among many possibilities,

$$\int_3^7(x+2)\sin(x-1)\,dx = \lim_{n\to\infty}\left\{\frac{4}{n}\sum_{i=1}^{n}\left(5+\frac{4}{n}i\right)\sin\left(2+\frac{4}{n}i\right)\right\}.$$

18. a. Since $(\sqrt{x}-3)^2 = x - 6\sqrt{x}+9$, it follows that

$$\int (\sqrt{x}-3)^2\,dx = \tfrac{1}{2}x^2 - 4x\sqrt{x} + 9x + C.$$

b. Writing $(\tan\vartheta - (\cos\vartheta)^2 + \sec\vartheta)/\cos\vartheta = \sec\vartheta\tan\vartheta - \cos\vartheta + \sec^2\vartheta$, gives

$$\int \frac{\tan\vartheta - \cos^2\vartheta + \sec\vartheta}{\cos\vartheta}\,d\vartheta = \sec\vartheta - \sin\vartheta + \tan\vartheta + C.$$

c. Integrating by inspection gives

$$\int_1^2 \left(e^y - \frac{3}{y^2}\right)dy = \left(e^y + \frac{3}{y}\right)\Big|_1^2 = e^2 - e - \tfrac{3}{2}.$$

d. If a lies in the domain of f, then $\int_a^a f = 0$, so the integral is zero.

19. Writing the given equation as

$$\int_0^y e^{-t}\,dt - \int_2^{x^2} \sin^2(t)\,dt = 4,$$

gives (by implicit differentiation and the fundamental theorem of calculus)

$$\frac{dy}{dx} = -\frac{-2x\sin^2(x^2)}{e^{-y}} = 2xe^y\sin^2(x^2).$$

---

20. – If $f(x) = x^2$ and $g(x) = x$, then

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0, \qquad \text{but} \qquad \lim_{x\to 0}\frac{f(x)}{g(x)} = \lim_{x\to 0} x = 0.$$

So the statement is false.

– By the continuity of differentiable functions, if f is differentiable at a then $\lim_{x\to a} f(x) = f(a)$. So the statement is true.

– The cube root function is increasing on $[-1,1]$ but not differentiable at 0, so the statement is false.

In fact, it is not difficult to define a function f which is differentiable and increasing on $[0,1]$ (*i.e.*, $f(x) < f(y)$ if $0 \leqslant x < y \leqslant 1$), and satisfies $f'(x) = 0$ for 99.999999999999999999999999999999999999999999999999999999 % of the values of x in $[0,1]$. (Doing so is left as an exercise.)

– The statement in is false, since

$$\int_0^1 x^2\,dx = \tfrac{1}{3}, \qquad \text{but} \qquad \int_0^1 x\,dx \cdot \int_0^1 x\,dx = \tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{4},$$

– If $f(x) = -1/x$, then $f'(x) = 1/x^2 > 0$ wherever $f(x)$ is defined (*i.e.*, where $x \neq 0$). However, $f(-1) = 1 > -1 = f(1)$, so f is not increasing on its domain. Therefore, the statement in question is false.