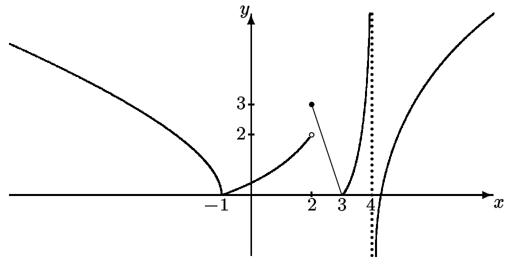
CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

(Mathematics 201–NYA/??)

1. Consider the following graph of $f(x)$.



   (a) Use the above graph to find the following:
   (i) $\lim_{x\to -1} f(x)$ (ii) $\lim_{x\to 2^-} f(x)$ (iii) $\lim_{x\to 2^+} f(x)$
   (iv) $\lim_{x\to 2} f(x)$ (v) $\lim_{x\to 4^-} f(x)$ (vi) $\lim_{x\to 4^+} f(x)$
   (b) Is $f(x)$ continuous at $x = 2$?
   (c) Is $f(x)$ differentiable at $x = -1$?

2. Evaluate any 3 of the following:
   (a) $\lim_{x\to 2} \dfrac{2x^2 - 3x - 2}{x^2 - x - 2}$ (b) $\lim_{x\to 4} \dfrac{\sqrt{25 - x^2} - 4}{x - 3}$
   (c) $\lim_{x\to -5^+} \dfrac{x^2 - 2x - 35}{x^2 + 10x + 25}$ (d) $\lim_{x\to 3} \dfrac{x - 3}{x^3 - 27}$

3. Let $f(x) = \begin{cases} 1 & \text{if } x < -1 \\ x^3 & \text{if } -1 \leqslant x < 2 \\ 3x + 2 & \text{if } x \geqslant 2. \end{cases}$
   (a) Sketch the graph of $f(x)$.
   (b) Discuss the continuity of $f(x)$ at $x = -1$ and $x = 2$.

4. A particle moves on a line from its initial position so that after t hours it is $s = 16t^2$ miles from its initial position.
   (a) Find the average velocity of the particle over the interval $[1, 3]$.
   (b) Find the instantaneous velocity at $t = 1$.

5. If $f(x) = \sqrt{x}$, use the limit definition of the derivative to find $f'(x)$.

6. (a) If $f(x) = \dfrac{\tan x}{1 + x\tan x}$, then find $f'(x)$ and simplify.
   (b) If $y = x^{1/3}(2x - 1)^{2/3}$, then find y' and simplify.

7. Find the equation of the tangent line to the curve $y = 3x^2 - 5x - 7$ at $x = 1$.

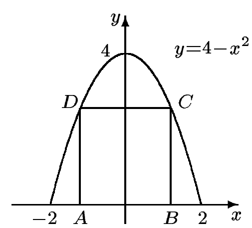
8. Find y'. Do not simplify your answers.
   (a) $y = \sin^4(3x + 1)$ (b) $y = \log(\cos x)$
   (c) $y = \dfrac{\sqrt{x^2 - 2}\sqrt[3]{x^3 + 4}}{(x - 6)^4}$ (Use logarithmic differentiation.)
   (d) $xy = y^3 + 4x^3$ (e) $y = e^{3x}(3x - 1)$

9. If $f(x) = \ln(\ln x)$, then find the exact value for $f''(e)$.

10. The length of a rectangle increases at a rate of 2 cm/sec while the width decreases at a rate of 3 cm/sec. Find the rate of change of the area when the length is 5 cm and the width is 4 cm.

11. Given: $f(x) = \dfrac{18(x - 1)}{x^2}$,
    $f'(x) = \dfrac{-18(x - 2)}{x^3}$ and $f''(x) = \dfrac{36(x - 3)}{x^4}$.
    (a) Find all intercepts, asymptotes, relative extrema and points of inflection of $f(x)$.
    (b) Use the information in (a) to sketch a large well-labeled graph of $f(x)$.

12. Find the values of x where the minimum and maximum values of $f(x) = \dfrac{x^2}{x + 3}$ occur on the interval $[-1, 1]$. State the minimum and maximum values of $f(x)$ on $[-1, 1]$.

13. 

    If the base AB must remain on the x-axis and, the points C and D must remain on the parabola, $y = 4 - x^2$, what are the dimensions of the rectangle, $ABCD$, with maximum area?

14. Evaluate (*No decimals*):
    (a) $\displaystyle\int (\sin x + e^x - 3^x)\,dx$ (b) $\displaystyle\int \frac{x^4 + 7x^2 + 5}{x^3}\,dx$
    (c) $\displaystyle\int_4^9 \left(\sqrt{x} - \frac{2}{\sqrt{x}} + x\right)dx$ (d) $\displaystyle\int_0^1 (x + 1)^2\,dx$
    (e) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \sec x(\tan x + \sec x)\,dx$

15. Find a function, y, such that $y'' = 2x$, $y'(1) = -1$, and $y(1) = 2$.

16. Find $f(x)$ if $\displaystyle\int f(x)\,dx = \sqrt{1 - x} + C$.

17. (a) Sketch the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$.
    (b) Find the area of the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$. (*No decimals*)