CEGEP JOHN ABBOTT COLLEGE

Calculus I (Science)

(Mathematics 201–NYA/??)

1. Evaluate:
   (a) $\displaystyle\lim_{x\to-\infty}\frac{4x+3}{3x^2-5}$ (b) $\displaystyle\lim_{x\to-1^-}\frac{x^2+2x}{x+1}$
   (c) $\displaystyle\lim_{x\to0}\frac{\sqrt{x^2+4}-2}{x}$ (d) $\displaystyle\lim_{x\to3}\frac{x^2-2x-3}{2x^3-5x^2-3x}$

2. Given the function $f(x)=\begin{cases}2-x & \text{if } x<5\\ x-8 & \text{if } x\geqslant 5.\end{cases}$
   (a) Sketch the graph of $f(x)$, writing x and y intercepts on the graph.
   (b) State the domain and range of $f(x)$.
   (c) Is $f(x)$ differentiable at $x=5$? Why or why not?

3. Given the function $f(x)=\begin{cases}-\dfrac{1}{x+2} & \text{if } x\leqslant-1\\ x^3-1 & \text{if } -1<x\leqslant 2\\ 9-x & \text{if } x>2.\end{cases}$
   Using the definition of continuity show that
   (a) $f(x)$ is continuous at $x=2$.
   (b) $f(x)$ is not continuous at $x=-1$.

4. Use the limit definition of the derivative to find $f'(x)$ for the function $f(x)=-\dfrac{3}{x}$.

5. Find y' but *do not simplify* your answer.
   (a) $y=e^4-\sqrt[3]{5x}-\dfrac{1}{3(x+1)}+\dfrac{3x}{5}$
   (b) $y=\ln\left((2x-1)^{1/3}(5x+2)^{2/3}(x-1)^{4/3}\right)$
   (c) $y=\tan(3x)-\sec\left(3x^5\right)-\sin^2\sqrt{x}$
   (d) $y=2^{x^2}-\log_2(3x-4)$
   (e) $x\cos y+y\cos x=1$ (f) $y=(\sin 3x)^{2x}$

6. Find y' and *simplify* as much as possible:
   (a) $y=e^{x\sqrt{1+x^2}}$ (b) $y=(3x+2)^2(1-5x)^3$
   (c) $y=\dfrac{x^2-x}{(x+1)^3}$

7. Find the equation of the tangent line to the curve $y=\cos(2x)$ at $x=\frac{\pi}{4}$.

8. If $f(x)=\dfrac{2\cos x}{1-\sin x}$, find $f''(\pi)$.

9. A 26-foot ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 3 feet per second, how fast will the bottom of the ladder be moving away from the wall when the top is 10 feet above the ground?

10. Given: $f(x)=\dfrac{2+x-2x^2}{(x-1)^2}$,
    $$f'(x)=\frac{3x-5}{(x-1)^3} \quad\text{and}\quad f''(x)=\frac{6(2-x)}{(x-1)^4}.$$
    (a) Find all asymptotes, relative extrema and points of inflection of $f(x)$.
    (b) State all intervals where $f(x)$ is increasing, decreasing, concave up and concave down.
    (c) Using the preceding information, sketch a large well-labeled graph of $f(x)$.

11. Find the absolute maximum and the absolute minimum of $f(x)=\dfrac{1-x}{x^2+3}$ on the interval $[-2,3]$.

12. A cylindrical metal can, open at the top, is to hold 1000 cm^3 of liquid. Find the height and radius so that a minimum amount of metal is needed to manufacture the can. (The volume of a cylinder with radius r and height h is πr^2h, and the surface area of its curved portion is $2\pi rh$).

13. Evaluate (*no decimals*):
    (a) $\displaystyle\int(\vartheta+\csc^2\vartheta-\sec^2\vartheta)\,d\vartheta$ (b) $\displaystyle\int_e^{e^2}\left(\frac{3}{x}+4\right)dx$
    (c) $\displaystyle\int_0^{\frac{\pi}{4}}\frac{\cos^3\vartheta+2\sin\vartheta}{\cos^2\vartheta}\,d\vartheta$ (d) $\displaystyle\int_1^8\frac{(x-1)^2}{\sqrt[3]{x}}\,dx$

14. Find $f(x)$ given that $f''(x)=x+\sqrt{x}$, $f(x)=1$ and $f'(1)=2$.

15. (a) Sketch the graph of $f(x)=x^3-1$.
    (b) Find the area enclosed by $f(x)=x^3-1$, the x-axis, $x=-2$ and $x=3$.