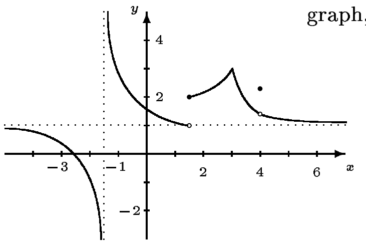
CEGEP JOHN ABBOTT COLLEGE

CALCULUS I (SCIENCE)

(MATHEMATICS 201–NYA/??)

Autumn 1996

1. The graph of a functon f is shown below. By referring to the graph, find the following values:



   (a) $f(1.5)$
   (b) $\lim_{x\to 3} f(x)$
   (c) $\lim_{x\to -\infty} f(x)$
   (d) $\lim_{x\to 1.5^-} f(x)$
   (e) $\lim_{x\to 1.5} f(x)$
   (f) $\lim_{x\to 4} f(x)$
   (g) One value of x for which $f'(x) > 0$.
   (h) One value of x for which $f(x)$ is continuous but $f'(x)$ does NOT exist.

2. Use a graphics calculator *or* numerical approximation to estimate $\lim_{x\to 1} \dfrac{\ln x}{x-1}$.

3. Evaluate the following limits *algebraically*:
   (a) $\lim_{t\to 4} \dfrac{t-4}{t^2-3t-4}$ (b) $\lim_{x\to 4} \sqrt{x+\sqrt{x}}$
   (c) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$ (d) $\lim_{x\to\infty} \dfrac{3x^2+x+5}{4x^2+4x+1}$

4. Consider the function

$$f(x) = \begin{cases} x & \text{if } x \leqslant 1 \\ 2-x & \text{if } 1 < x < 2 \\ 2+x & \text{if } x \geqslant 2. \end{cases}$$

   Use the definition of continuity at a point to:
   (a) determine whether or not f is continuous at $x = 1$.
   (b) determine whether or not f is continuous at $x = 2$.

5. Use a limit definition of the derivative to find $f'(x)$ for $f(x) = \dfrac{2}{x-1}$.

6. Find the derivatives of the following functions. DO NOT simplify your answers.
   (a) $y = 5x^{1/10} + \dfrac{8}{\sqrt[4]{x}} + e^{\sqrt{\pi}}$ (b) $g(x) = \dfrac{\sqrt{x+1}}{\sqrt{x}+1} + \sec^3(5x)$
   (c) $f(x) = e^{\tan x}\ln x$ (d) $y = 5^x + \log_3 x$
   (e) $y = \sin(\pi x + 1) + \cos(5x)$

7. Let $f(x) = \dfrac{\cos(x-1)}{x+1}$. Find an equation of the line tangent to the graph of $y = f(x)$ at the point $\left(1, \frac{1}{2}\right)$.

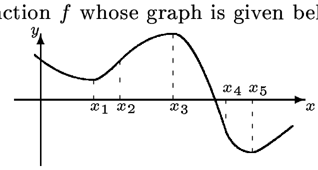
8. Given $x^3 + y^3 = 6xy + 1$. (a) Find $\dfrac{dy}{dx}$. (b) Evaluate $\dfrac{dy}{dx}$ at $(1, 0)$.

9. Use logarithmic differentiation to calculate $\dfrac{dy}{dx}$. Express your answer in terms of x *alone*. DO NOT simplify.
   (a) $y = \dfrac{(2x+1)^3(2x-3)^2}{(x^2+2)^4}$ (b) $y = (x+1)^x$

10. Let $f(x) = 4 - 3x^{2/3} + 2x$, $-1 \leqslant x \leqslant 2$. Given that f is continuous throughout the interval $[-1, 2]$, the Extreme Value Theorem guarantees that f attains an absolute maximum and an absolute minimum somewhere in that interval. Find them *algebraically*.

11. Sketch the graph of $y = xe^x$, showing intercepts, relative extrema and inflection point(s). *Show all supporting work.*

12. Consider the function f whose graph is given below.



   (a) At what value(s) of x does $f'(x)$ change sign?
   (b) At what value(s) of x does $f'(x)$ have a local (relative) maximum?
   (c) At what value(s) of x does $f'(x)$ have a local (relative) minimum?
   (d) Sketch a graph of $f'(x)$.
   (e) At what value(s) of x does $f''(x)$ change sign?

13. A winch situated on a dock at a point 8 meters above the water level is reeling in a cable at a constant rate of 5 meters per minute.
   (a) If z (in meters) is the length of cable between the winch and the surface of the water, express z as a function of the angle ϑ (in radians) the cable makes with the water.
   (b) Calculate the rate at which the angle ϑ is changing at the instant when $z = 24$ meters. Give your answer correct to 2 decimal places.

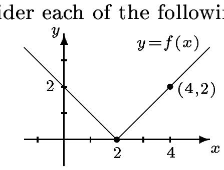
14. A rectangular box with no top and a square base is to have a fixed volume of 16,000 cm^3. Find the dimensions of the box with the least surface area.

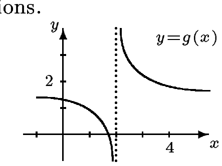
15. Find the following integrals:
   (a) $\displaystyle\int \left(\frac{x^4}{3} - \frac{4}{x^3} + \sqrt[4]{x^3} - \pi^4\right) dx$ (b) $\displaystyle\int_1^3 \left(x + \frac{1}{x}\right)^2 dx$
   (c) $\displaystyle\int \frac{\cos^2\vartheta + 1}{\cos^2\vartheta}\, d\vartheta$ (d) $\displaystyle\int \left(5e^x + xe^5 - 5\cos x\right) dx$

16. Find the area bounded by the graphs of $y = \sec x \tan x + \sin x$ and $y = 0$ from $x = 0$ to $x = \frac{\pi}{3}$.

DO ONLY ONE PROBLEM OF QUESTIONS 17, 18, OR 19.

17. Consider each of the following functions.





   Fill in the table below:

Function	Derivative from the left of $x=2$	Derivative from the right of $x=2$	Derivative at $x=2$
$f(x)$			
$g(x)$			

18. A graphics calculator is required for this problem.
   (a) Use your graphics calculator to sketch the graph of $f'(x)$ if $f'(x) = (\ln x)^2 - 2(\sin x)^4$ for $0 < x \leqslant 7.5$.
   (b) By using the graph of $f'(x)$, state:
      (i) where $f(x)$ is increasing and where $f(x)$ is decreasing.
      (ii) where $f(x)$ is concave up and where $f(x)$ is concave down.

19. Consider a function $g(t)$ (representing distance in centimeters and t representing time in seconds) that has led to the following numerical output when $t = 6$ seconds.

Δt	-3	-0.3003	-0.1583	-0.0001
$\dfrac{g(t+\Delta t) - g(t)}{\Delta t}$	0.2310	0.7098	0.1668	0.1667

   (a) What is the average velocity between $t = 3$ seconds and $t = 6$ seconds?
   (b) From the numerical output, can you estimate the right or left hand derivative of $g(t)$ when $t = 6$ seconds? Estimate the value of this derivative.
   (c) If the right and left hand derivatives of $g(t)$ are equal, what is the instantaneous velocity of $g(t)$ when $t = 6$ seconds?