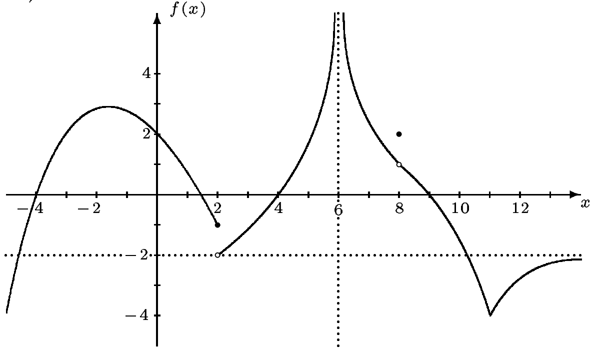
CEGEP JOHN ABBOTT COLLEGE

Winter 1997

Calculus I (Science)

(Mathematics 201–NYA/??)

1. Refer to the sketch below to answer the following quesitons. If a limit does not exist, state in which way (∞, $-\infty$, or "does not exist").



(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$
(d) $\lim_{x\to 6^-} f(x)$ (e) $\lim_{x\to 6^+} f(x)$ (f) $\lim_{x\to 6} f(x)$
(g) $\lim_{x\to 8} f(x)$ (h) $f(8)$ (i) $\lim_{x\to -\infty} f(x)$
(j) $\lim_{x\to \infty} f(x)$ (k) $\lim_{x\to 11} f(x)$
(l) Give a value of x for which f is continuous and *at the same time* f is not differentiable.

2. Use algebraic techniques to calculate the following limits. If an answer is undefined, assign the symbol $+\infty$ or $-\infty$ if possible.

(a) $\lim_{x\to 2} \dfrac{\sqrt{x^2+5}+1}{x-1}$ (b) $\lim_{x\to 2} \dfrac{4|x-2|}{x-2}$ (c) $\lim_{x\to\infty} \dfrac{2x^2-5x-7}{x^2-5}$

(d) $\lim_{x\to 2} \dfrac{3x^2-8x+4}{x-2}$ (e) $\lim_{x\to 2^+} \dfrac{3x^2-8x+4}{(x-2)^2}$

3. (a) Compute the table below for $f(x) = \dfrac{e^{2x}-1}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

(b) Use your results to give $\lim_{x\to 0} \dfrac{e^{2x}-1}{x}$.

4. Given: $f(x) = \begin{cases} x^3 & \text{if } x \leqslant -1 \\ x^2+1 & \text{if } -1 < x \leqslant 1 \\ 3-x & \text{if } x > 1 \end{cases}$

Use the definition of continuity at a point to:
(a) determine whether or not f is continuous at $x=-1$.
(b) determine whether or not f is continuous at $x=1$.

5. Given: $g(x) = \dfrac{2x^2-3x-5}{x^2-1}$
(a) Find the equations for all vertical asymptotes for $g(x)$.
(b) Find any other discontinuities for $g(x)$.

6. Find a value for k which will make f continuous at $x=-1$.

$$f(x) = \begin{cases} 3-2x & \text{if } x \leqslant -1 \\ kx^2+2 & \text{if } x > -1 \end{cases}$$

7. Consider the formula $s(t) = t^2+t$ in which t is measured in seconds and $s(t)$ is measured in centimeters.
(a) Determine the average velocity from $t=3$ to $t=4$.
(b) *Use a limit definition* to find the derivative $s'(t)$.
(c) What is the instantaneous velocity at $t=3$?

8. Give the derivatives of the following functions. *Do not simplify* your answers.

(a) $f(x) = 4x^3 - \dfrac{2}{5x^2} + \dfrac{3}{e^x} - \ln\sqrt{2}$ (b) $g(t) = \dfrac{\ln(1+t^2)}{\sqrt{1+t^2}}$

(c) $y = \sin(2x)\sec(3x)$ (d) $y = x^{x^2+x}$

(e) $w = \cos^3\left(\dfrac{\vartheta}{2}\right)$ (f) $y = \dfrac{\sqrt{x}\tan^2 x}{(4+9x^2)^{\frac{1}{3}}}$

(Use logarithmic differentiation.)

9. Find the second-order derivative $f''(x)$ for $f(x) = e^{x-e^x}$. *Do not simplify.*

10. Find all x-values at which the graph of f has a horizontal tangent line. $f(x) = 4x + \dfrac{1}{x} - 2$.

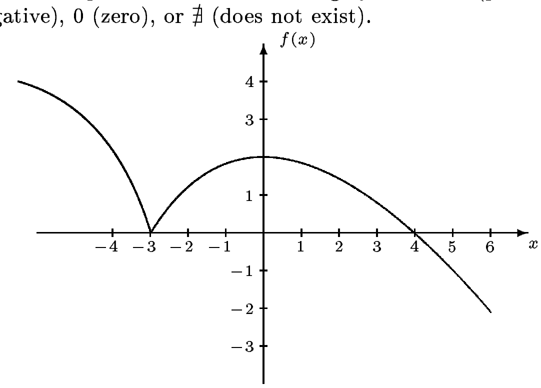
11. Given $x^2y + y^2x = -2$. (a) Find $\dfrac{dy}{dx}$. (b) Find an equation of the tangent line at $(-1,-1)$.

12. Find the maximum and minimum values of f on the given closed interval and state where these values occur. Justify your answer. $f(x) = \dfrac{x^2}{x^2+3}$ on $[-1,2]$.

13. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

14. Sketch the graph of a function f having the following charicteristics.
- $f(-5) = f(0) = f(4) = 0$
- $f'(x) < 0$ for $x < -3$ and for $3 < x < 5$
- $f'(x) > 0$ for $-3 < x < 3$ and for $x > 5$
- $f''(x) > 0$ for $x < 1$
- $f''(x) < 0$ for $1 < x < 3$
- $\lim_{x\to 5} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = 3$

15. By referring to the graph, complete the chart below by writing in each blank space one of the following symbols: $+$ (positive), $-$ (negative), 0 (zero), or $\nexists$ (does not exist).



	$x < -3$	$x = -3$	$-3 < x < 0$	$x = 0$	$0 < x < 4$
$f(x)$					
$f'(x)$					
$f''(x)$					

16. Evaluate the following integrals.

(a) $\displaystyle\int \left(2^3 - 2x^2 + \frac{1}{2\sqrt{x}}\right) dx$ (b) $\displaystyle\int (3 - \sec x \tan x)\, dx$

(c) $\displaystyle\int (2\cos x - e^x + e^2)\, dx$ (d) $\displaystyle\int \frac{(x+2)(2x-1)}{x^2}\, dx$

(e) $\displaystyle\int_0^{\frac{\pi}{4}} (\sec^2 x - \sin x)\, dx$

17. Find the area of the region bounded by the graph of $y = 4 - x^2$ and the x-axis.