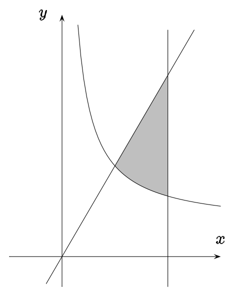
CEGEP JOHN ABBOTT COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/∞)

1. Let R be the region (as shown) bounded by $y = \dfrac{2}{x} + 1$, $y = 3x$, and $x = 2$.
   (a) Find the area of R.
   (b) Find the exact volume of the solid that results from revolving R about the y-axis.
   (c) Set up the integral required to find the volume of the solid that results from revolving R about the x-axis. *Do not actually evaluate the integral.*



2. If $y = x \arctan\left(\frac{1}{x}\right)$ find y'. (Do not simplify.)

3. Evaluate the following integrals.
   (a) $\displaystyle\int_1^e 2x \ln\sqrt{x}\, dx$
   (b) $\displaystyle\int \frac{2 + \sin\vartheta - \cos\vartheta \sin\vartheta}{\cos^2\vartheta}\, d\vartheta$
   (c) $\displaystyle\int \frac{x^3}{\sqrt{16 + x^2}}\, dx$
   (d) $\displaystyle\int_0^{\pi/3} \sin^3 3x \cos^2 3x\, dx$
   (e) $\displaystyle\int \frac{2x^2 - 10x + 7}{(2x+1)(x-2)^2}\, dx$
   (f) $\displaystyle\int 2x\sqrt{x+3}\, dx$

4. Calculate the following limits.
   (a) $\displaystyle\lim_{x\to 0} \frac{xe^x - x}{1 - \cos 2x}$
   (b) $\displaystyle\lim_{x\to 0^+} (1 - 2x)^{1/x^2}$
   (c) $\displaystyle\lim_{x\to\infty} \frac{\ln(x^4 + 1)}{\ln(x + 1)}$

5. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.
   (a) $\displaystyle\int_0^1 \frac{x\, dx}{\sqrt{1 - x^2}}$
   (b) $\displaystyle\int_1^\infty \frac{dx}{(1 + 2x)^3}$

6. Fnd the solution of the differential equation
$$xy\frac{dy}{dx} = x + 1, \quad x > 0, \quad y(1) = -2.$$

7. For the sequence $\{a_k\} = \left\{\dfrac{\cos(k\pi)}{e^k}\right\}$, determine whether or not it is convergent. (Justify your answer.)

8. Calculate (if possible) the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$.

9. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{n!2^n}{(2n)!}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$
   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{2n}\right)^n$
   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{2n+1}}{n^2 + 1}$

10. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)
    (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$
    (b) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{e^n}$

11. Determine the interval of convergence of the series
$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{4^{n+1}\sqrt[3]{n+1}}.$$

12. For the function $f(x) = \dfrac{1}{x+1}$
    (a) find the first five terms of the Maclaurin series for $f(x)$,
    (b) find the n^{th} term, and express the series in Σ-notation.
    (c) What is the radius of convergence for this series?

ANSWERS

1. (a) $A = \frac{7}{2} - 2\ln 2$
   (b) $V = 2\pi \int_1^2 x(3x - \frac{2}{x} - 1)\, dx = 7\pi$
   (c) $V = \pi \int_1^2 (3x)^2 - (\frac{2}{x} + 1)^2\, dx$
2. $y' = \arctan\left(\frac{1}{x}\right) - \dfrac{x}{x^2 + 1}$
3. (a) $\frac{1}{4}(e^2 + 1)$
   (b) $2\tan\vartheta + \sec\vartheta - \ln|\sec\vartheta| + C$
   (c) $\frac{1}{3}\sqrt{16 + x^2}(x^2 - 32) + C$
   (d) $\frac{4}{45}$
   (e) $\ln|2x + 1| + \dfrac{1}{x - 2} + C$
   (f) $\frac{4}{5}(x+3)^{3/2}(x - 2) + C$
4. (a) $\frac{1}{2}$ (b) 0 (c) 4
5. (a) The integral converges to 1.
   (b) The integral converges to $\frac{1}{36}$.
6. $y = -\sqrt{2x + 2\ln x + 2}$
7. The sequence converges to 0.
8. $\frac{3}{2}$
9. (a) Convergent (RaT)
   (b) Divergent (the terms approach $e \neq 0$)
   (c) Convergent (RoT)
   (d) Convergent (lim. comp. with $\sum n^{-3/2}$)
10. (a) Cond. conv. ($\int$-test & AST)
    (b) Abs. conv. (comp. with $\sum e^{-n}$)
11. $-6 \leqslant x < 2$
12. The Maclaurin series is $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$, and the radius of convergence 1.