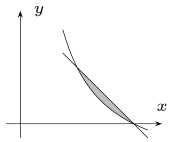
CEGEP
JOHN ABBOTT
COLLEGE

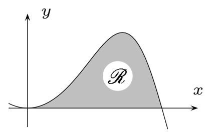
CALCULUS II (SCIENCE)

(MATHEMATICS 201-NYB/∞)

1. Find the area of the region between the curves
$$y = \frac{2}{x} - 1 \quad \text{and} \quad y = 2 - x.$$



2. Let $\mathscr{R}$ be the region bounded by $y = \sin\left(x^2\right)$, $y = 0$, and $x = \sqrt{\pi}$.
   (a) Find the exact volume of the solid that results from revolving $\mathscr{R}$ about the y-axis.
   (b) Set up the integral required to find the volume of the solid that results form revolving the $\mathscr{R}$ about the x-axis. *Do not evaluate the integral*.



3. If $y = \dfrac{\operatorname{arcsec}\sqrt{x}}{e^{2x}}$, find y'. (Do not simplify your answer.)

4. Evaluate the following integrals.
   (a) $\displaystyle\int e^{2x}\sin x\,dx$ (b) $\displaystyle\int \frac{2}{t^2}\left(1 - \frac{1}{t}\right)^2 dt$
   (c) $\displaystyle\int \frac{dx}{\sqrt{1-16x^2}}$ (d) $\displaystyle\int \sin^2 3x\cos^2 3x\,dx$
   (e) $\displaystyle\int \frac{8x^2-3x-4}{(4x-1)(x^2+1)}\,dx$ (f) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x^2+1}}$

5. Calculate the following limits.
   (a) $\displaystyle\lim_{x\to1}\frac{x-e^{x-1}}{(x-1)^2}$ (b) $\displaystyle\lim_{x\to\infty}\left(\frac{x-1}{x+2}\right)^x$
   (c) $\displaystyle\lim_{x\to0^+}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$

6. Determine whether these improper integrals converge or diverge; if an integral converges, give the exact value of the integral.
   (a) $\displaystyle\int_0^1 \frac{x+1}{(x^2+2x)^{5/4}}\,dx$ (b) $\displaystyle\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

7. Find the solution of the differential equation
$$(x+1)e^y\frac{dy}{dx} = x, \quad x \geqslant 0,\ y(0) = 1.$$

8. Determine whether or not the sequence
$$\left\{(-1)^k\cos\left(\frac{1}{k}\right)\right\}$$
is convergent. (Justify your answer.)

9. Calculate (if possible) the sum of the series $\displaystyle\sum_{n=1}^\infty \frac{1}{2n^2+2n}$.

10. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)
    (a) $\displaystyle\sum_{n=0}^\infty \frac{(n!)^2}{(2n)!}$ (b) $\displaystyle\sum_{n=1}^\infty\left(\frac{1}{n} - \frac{1}{n^2}\right)$
    (c) $\displaystyle\sum_{n=1}^\infty\left(\frac{1}{n} - \frac{1}{n^2}\right)^n$ (d) $\displaystyle\sum_{n=1}^\infty \frac{\sqrt{n^3-1}}{n^2+1}$

11. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)
    (a) $\displaystyle\sum_{n=0}^\infty(-1)^n\frac{\arctan n}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^\infty(-1)^n\cos\left(\frac{1}{n}\right)$

12. Determine the interval of convergence of the series
$$\sum_{n=1}^\infty \frac{3^{n-1}(x+1)^n}{n\sqrt{n+1}}.$$

13. For the function $f(x) = e^{2x}$
    (a) find the first five terms of the Maclaurin series of $f(x)$;
    (b) find the n^{th} term, and express the series in Σ-notation.
    (c) What is the radius of convergence of this series.

ANSWERS

1. $A = \frac{3}{2} - 2\ln 2$
2. (a) 2π (b) $\displaystyle\pi\int_0^{\sqrt{\pi}}\sin^2\left(x^2\right)\,dx$
3. $y' = \dfrac{\dfrac{e^{2x}}{2x\sqrt{x-1}} - 2e^{2x}\operatorname{arcsec}\sqrt{x}}{e^{4x}}$
4. (a) $\frac{1}{5}e^{2x}(2\sin x - \cos x) + C$ (b) $\frac{2}{3}\left(1-\frac{1}{t}\right)^3 + C$
   (c) $\frac{1}{4}\arcsin 4x + C$ (d) $\frac{1}{8}x - \frac{1}{96}\sin 12x + C$
   (e) $-\ln|4x-1| + \frac{3}{2}\ln(x^2+1) + C$ (f) $\ln(1+\sqrt{2})$
5. (a) $-\frac{1}{2}$ (b) e^{-3} (c) 0
6. (a) The integral diverges to ∞ (b) The integral converges to $\frac{2}{e}$
7. $y = \ln(x - \ln(x+1) + e)$
8. The sequence diverges (oscillates); the terms with even index tend to 1 and the terms with odd index tend to -1.
9. The sum of the series is $\frac{1}{2}$.
10. (a) The series converges by the ratio test: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2(2n+1)} \to 0$.
    (b) The series diverges by the limit comparison test (with $\sum\frac{1}{n}$).
    (c) The series converges by the root test: $\sqrt[n]{|a_n|} = \frac{1}{n} - \frac{1}{n^2} \to 0$.
    (d) The series diverges by the limit comparison test (with $\sum\frac{1}{\sqrt{n}}$).
11. (a) The series is absolutely convergent by the integral test, or the comparison test (with $\sum n^{-2}$).
    (b) The series diverges by the vanishing criterion (*i.e.*, its general term does not converge to zero; *cf.* question 8).
12. The interval of convergence is $\left[-\frac{4}{3}, -\frac{2}{3}\right]$.
13. The Maclaurin series is
$$\underbrace{1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \cdots}_{\text{(a)}} = \underbrace{\sum_{n=0}^\infty \frac{2^nx^n}{n!}}_{\text{(b)}}.$$
    (c) The series has infinite radius of convergence.