CEGEP
JOHN ABBOTT
COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201-NYB/∞)

1. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{9}{(x-1)^2(2x+1)}\,dx$ (b) $\displaystyle\int \frac{dx}{x^2\sqrt{49-x^2}}$

   (c) $\displaystyle\int \sqrt{\tan 3x}\sec^4 3x\,dx$ (d) $\displaystyle\int \sin 2x\cos 3x\,dx$

   (e) $\displaystyle\int_0^{\frac{1}{3}} \frac{\arctan 3x}{1+9x^2}\,dx$ (f) $\displaystyle\int_0^5 \frac{x\,dx}{\sqrt{3x+1}}$

   (g) $\displaystyle\int \frac{3x^2+3x+2}{x^2+1}\,dx$ (h) $\displaystyle\int x\operatorname{arcsec} x\,dx$

2. Calculate the following limits.

   (a) $\displaystyle\lim_{x\to\infty}(e^x+x)^{2/x}$ (b) $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x-1}{\cos 2x+\sin x}$

3. Determine whether the following integrals converge or diverge. If an integral converges, give the exact value.

   (a) $\displaystyle\int_e^{\infty} \frac{dx}{x(\ln x)^2}$ (b) $\displaystyle\int_0^3 \frac{dx}{(x-1)^{4/3}}$

4. Find the solution of the differential equation

$$2ye^x\frac{dy}{dx}-x=0,\ \ y(0)=2.$$

5. Consider the sequence $\{a_n\}=\left\{\dfrac{n^2+2}{2n(n+1)}\right\}$.

   (a) Does the sequence converge and if so, to what value?

   (b) Does the corresponding series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge? (Justify your answer.)

6. Determine whether the following series converge or diverge. State the test you are using and display a proper solution.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5n-1}{3n+2}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{2^n}-\frac{1}{n\sqrt{n}}\right)$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{|\sin n|}{n^2}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{\sqrt{n}}}{\sqrt{n}}$

7. Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n!}{n^2 3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{3n-1}}$

8. Given the series $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n-1}}{5^{n+1}}$,

   (a) find a formula for s_n, the n^{th} partial sum of the series,
   (b) find the sum of the series,

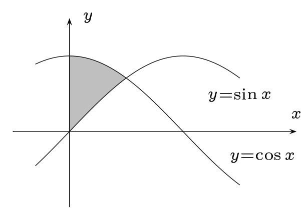
9. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{3^n(x-2)^{n+1}}{2n+1}.$$

10. (a) Find the first three non-zero terms of the Taylor series expansion of $f(x)=\sin x$ centred at $x=\pi/2$.
    (b) Use Σ-notation to write the general form of the series in (a).

11. (a) Sketch the region $\mathscr{R}$ bounded by $y^2=x+2$ and $y=x$.
    (b) Find the area of $\mathscr{R}$.

12. Let $\mathscr{R}$ be the shaded region bounded by $y=\cos x$, the y-axis and $y=\sin x$.



    (a) *Set up the integral(s)* needed to find the volume of the solid of revolution that results when $\mathscr{R}$ is rotated about
        (i) the x-axis, (ii) the y-axis.
    (b) Evaluate *one* of the integrals (i) or (ii). (*Not both.*)

ANSWERS

1. (a) $-2\ln|x-1|-\dfrac{3}{x-1}+2\ln|2x-1|+C$ (b) $-\dfrac{\sqrt{49-x^2}}{49x}+C$

   (c) $\frac{2}{9}(\tan 3x)^{3/2}+\frac{2}{21}(\tan 3x)^{7/2}+C$ (d) $\frac{1}{2}\cos x-\frac{1}{10}\cos 5x+C$

   (e) $\frac{\pi^2}{96}$ (f) 4

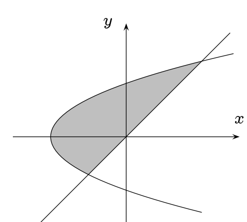
   (g) $3x+\frac{3}{2}\ln(x^2+1)-\arctan x+C$ (h) $\frac{1}{2}(x^2\operatorname{arcsec} x-\sqrt{x^2-1})+C$

2. (a) e^2 (b) $-\frac{2}{3}$
3. (a) The integral converges to 1. (b) The integral diverges.
4. $y^2=5-(x+1)e^{-x}$.
5. (a) The sequence converges to $\frac{1}{2}$.
   (b) The series diverges by the vanishing criterion.
6. (a) The series diverges by the root test $\left(\sqrt[n]{|a_n|}\to\frac{5}{3}\right)$.
   (b) The series converges because it is the difference of two convergent series (the first is geometric with $r=\frac{1}{2}$, and the second is a p-series with $p=\frac{3}{2}$).
   (c) The series converges by the comparison test (with $\sum 1/n^2$).
   (d) the series diverges by the vanishing criterion (the terms are greater than 1), or by the comparison test (with $\sum 1/\sqrt{n}$), or by the integral test (although the integral test *as stated* requires the function to be decreasing).
7. (a) The series diverges by the ratio test $\left(\left|\frac{a_{n+1}}{a_n}\right|\to\infty\right)$.
   (b) The series is conditionally convergent; convergent by the alternating series test, and not absolutely convergent by the limit comparison test (with $\sum 1/\sqrt{n}$).
8. (a) $s_n=\frac{1}{10}\cdot\left\{1-\left(\frac{3}{5}\right)^n\right\}$ (b) The sum of the series is $\frac{1}{10}$.
9. The radius of convergence is $\frac{1}{3}$ and the interval of convergence is $\left[\frac{5}{3},\frac{7}{3}\right)$.
10.

$$\sin x=\underbrace{1-\frac{1}{2}(x-\tfrac{\pi}{2})^2+\tfrac{1}{24}(x-\tfrac{\pi}{2})^4}_{\text{(a)}}-\cdots$$

$$=\underbrace{\sum_{n=0}^{\infty}\tfrac{(-1)^n}{(2n)!}(x-\tfrac{\pi}{2})^{2n}}_{\text{(b)}}$$

11. (a)



    (b) The area is $4\frac{1}{2}$.

12. The volume (i) is $\displaystyle\pi\int_0^{\frac{\pi}{4}}\cos^2 x-\sin^2 x\,dx=\frac{\pi}{2}$, and the volume (ii) is $\displaystyle 2\pi\int_0^{\frac{\pi}{4}}x(\cos x-\sin x)\,dx=\frac{\pi^2}{\sqrt{2}}-2\pi$.