CEGEP JOHN ABBOTT COLLEGE

Calculus II (Science)

(Mathematics 201-NYB/∞)

1. Evaluate the following.
   (a) $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$ (b) $\displaystyle\int_0^4 (t-1)\sqrt{1+2t}\,dt$
   (c) $\displaystyle\int (1+\sin x)^2\,dx$ (d) $\displaystyle\int \tan^3\left(\frac{x}{2}\right)\sec^4\left(\frac{x}{2}\right)\,dx$
   (e) $\displaystyle\int \frac{\sqrt{4x^2-9}}{x}\,dx$ (f) $\displaystyle\int \sqrt{x}\ln x\,dx$
   (g) $\displaystyle\int \frac{8x^3-6x^2+3x-4}{x^2(x^2+1)}\,dx$
2. Calculate the following limits.
   (a) $\displaystyle\lim_{x\to 0}\frac{x-\sin x}{x-\tan x}$ (b) $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$
3. Determine whether the following integrals converge or diverge. If an integral converges, give its exact value.
   (a) $\displaystyle\int_0^{\infty} xe^{x^2}\,dx$ (b) $\displaystyle\int_0^9 \frac{1+\sqrt{x}}{\sqrt{x}}\,dx$
4. Find the solution, y, for $(x^2+1)y' = xy$ when $y > 0$ and $y(0) = 1$.
5. $\mathscr{R}$ is the region bounded by $y = \dfrac{8}{x}$ and $y = 6 - x$.
   (a) Find the exact value of the area of $\mathscr{R}$.
   (b) Find the volume of the solid of revolution when $\mathscr{R}$ is rotated about the y-axis.
   (c) Set up the definite integral that represents the volume of the solid of revolution that results when $\mathscr{R}$ is rotated about the x-axis.
6. Consider the sequence $\{a_n\} = \left\{\dfrac{3n-1}{4n+3}\right\}$
   (a) Does the sequence converge, and if so, to what value?
   (b) Does the corresponding series $\displaystyle\sum_{n=0}^{\infty} a_n$ converge? *Justify your answer.*
7. Given the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^{n+1}}{7^n}$.
   (a) Find the formula for s_n, the n^{th} partial sum of the series.
   (b) Use the formula in part (a) to find the sum of the series.
8. Determine whether the following series converge or diverge. State which test you are using and display a proper solution.
   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$ (b) $\displaystyle\sum_{n=0}^{\infty}\frac{n^n}{2^n n!}$
   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n+2}{\sqrt{3n+1}}$ (d) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$
9. Determine whether the following series are absolutely convergent, conditionally convergent or divergent.
   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5^{n+1}}{7^n}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{7n+2}{3n-1}\right)^n$
   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+20}$
10. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n+1}(x+1)^n}{5^n\sqrt{3n+1}}$.
11. Given $f(x) = \ln x$
    (a) find the first 3 nonzero terms of the Taylor's series expansion of $f(x)$ centred at $a = 1$,
    (b) use Σ-notation to write the general form of the series in (a).

Answers

1. (a) $\frac{\pi^2}{32}$ (b) $\frac{56}{5}$
   (c) $\frac{3}{2}x - 2\cos x - \frac{1}{4}\sin 2x + C$ (d) $\frac{1}{2}\tan^4\left(\frac{x}{2}\right) + \frac{1}{3}\tan^6\left(\frac{x}{2}\right) + C$
   (e) $\sqrt{4x^2-9} - 3\,\mathrm{arcsec}\left(\frac{2x}{3}\right) + C$ (f) $\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$
   (g) $3\ln|x| + \frac{4}{x} + \frac{5}{2}\ln(x^2+1) - 2\arctan x + C$
2. (a) $-\frac{1}{2}$ (b) e^6
3. (a) The integral diverges. (b) The integral converges to 15.
4. $y = \sqrt{x^2+1}$
5. (a) $6 - 8\ln 2$ (b) $\frac{8\pi}{3}$
   (c) $V = \pi\displaystyle\int_2^4 (6-x)^2 - \left(\tfrac{8}{x}\right)^2\,dx$
6.
   (a) The sequence converges.
   (b) The series diverges by the vanishing criterion.
7. (a) $s_n = \frac{4}{9}\cdot\left\{1 - \left(-\frac{2}{7}\right)^n\right\}$ (b) $\displaystyle\lim_{n\to\infty} s_n = \frac{4}{9}$
8. (a) The series converges by the limit comparison test (with $\sum 1/n^2$).
   (b) The series diverges by the ratio test $\left(\left|\frac{a_{n+1}}{a_n}\right| \to \frac{e}{2}\right)$.
   (c) The series diverges by the comparison test (with $\sum 1/\sqrt{3n+1}$, which diverges by the limit comparison test or the integral test).
   (d) The series converges by the integral test.
9. (a) The series is absolutely convergent by the ratio test $\left(\left|\frac{a_{n+1}}{a_n}\right| = \frac{5}{7}\right)$.
   (b) The series diverges by the root test $\left(\sqrt[n]{|a_n|} \to \frac{7}{3}\right)$.
   (c) The series is conditionally convergent; it converges by the alternating series test, but not absolutely, by the limit comparison test (with $\sum 1/\sqrt{n}$).
10. The radius of convergence is $\frac{5}{2}$ and the interval of convergence is $\left[-\frac{7}{2}, \frac{3}{2}\right)$.
11.

$$\ln x = \underbrace{(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3}_{\text{(a)}} - \cdots$$

$$= \underbrace{\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-1)^n}{n}}_{\text{(b)}}$$