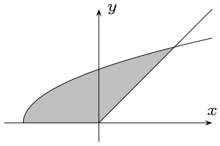
CALCULUS II (SCIENCE)

(MATHEMATICS 201-NYB/∞)

1. Differentiate and simplify $y = \dfrac{\arctan(e^x)}{1+e^{2x}}$.

2. Let $\mathscr{R}$ represent the region bounded by the graphs of $y = x$, the x-axis and $y^2 = x + 2$.



   (a) Find the area of $\mathscr{R}$.
   (b) Find the volume of the solid of revolution obtained when $\mathscr{R}$ is revolved about the x-axis.

3. Evaluate the following integrals. Give exact values for the definite integrals—*no decimals*.

   (a) $\displaystyle\int_1^5 x\sqrt{2x-1}\,dx$ (b) $\displaystyle\int_{5\sqrt{2}}^{10} \frac{dx}{x^3\sqrt{x^2-25}}$

   (c) $\displaystyle\int \frac{x+3}{(x+1)^2(3x+1)}\,dx$ (d) $\displaystyle\int_0^{\frac{\pi}{2}} e^{2x}\sin 5x\,dx$

   (e) $\displaystyle\int \cot^5 x\csc^3 x\,dx$ (f) $\displaystyle\int \frac{\sqrt{64-x^2}}{x^2}\,dx$

4. Evaluate the following limits.

   (a) $\displaystyle\lim_{x\to 0^+}(1+5x)^{\csc x}$ (b) $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$ (c) $\displaystyle\lim_{x\to\infty}\frac{(\ln x)^2}{x}$

5. Determine whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_0^1 x\ln x\,dx$ (b) $\displaystyle\int_{\frac{1}{2}}^{\infty}\frac{dx}{(2x-1)^{1/3}}$

6. Find the solution of the differential equation

$$(x^2+1)\frac{dy}{dx} = \frac{1}{ye^y}; \quad y(1) = 0.$$

7. Consider the sequence $\{a_n\} = \left\{(-1)^{n+1}\left(\dfrac{n+1}{2n+1}\right)\right\}$.

   (a) Does the sequence converge or diverge? (Justify your answer.)
   (b) Does the corresponding series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge or diverge? (Justify your answer.)

8. Determine whether the following series converge or diverge. State the test used and show that the conditions for applying the test have been met.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n+1)!}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{-\sqrt{n}}}{\sqrt{n}}$

   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(\frac{1+3n}{2+5n}\right)^n$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{2n+3}{n^4+2}$

9. Determine whether the following series are absolutely convergent, conditionally convergent or divergent. State the test used and show that the conditions for applying the test have been met.

   (a) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(n+3)}{n2^n}$

10. Find the radius and interval of convergence of the series

$$\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}(x-1)^n}{2^n}$$

11. (a) Find the first three nonzero terms of the Maclaurin series of $\cos x$.
    (b) Use sigma notation to write the general form of the series in (a).

12. Find the sum of the series.

    (a) $\displaystyle\sum_{n=1}^{\infty}3\left(-\frac{2}{5}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{4}{(n+3)(n+4)}$

ANSWERS

1. $\dfrac{dy}{dx} = \dfrac{e^x\{1-2e^x\arctan(e^x)\}}{(1+e^{2x})^2}$
2. (a) $10/3$ square units, (b) $16\pi/3$ cubic units.
3. (a) $428/15$, (b) $(\pi+3\sqrt{3}-6)/3000$, (c) $2\ln\left|\dfrac{3x+1}{x+1}\right| + \dfrac{1}{x+1} + C$,
   (d) $(2e^{\pi}+5)/29$, (e) $-\frac{1}{7}\csc^7 x + \frac{2}{5}\csc^5 x - \frac{1}{3}\csc^3 x + C$,
   (f) $-\dfrac{\sqrt{64-x^2}}{x} - \arcsin\left(\dfrac{x}{8}\right) + C$.
4. (a) e^5, (b) $-\frac{1}{2}$, (c) 0.
5. (a) The integral converges to $-\frac{1}{4}$. (b) The integral diverges.
6. $(y-1)e^y = \arctan x - 1 - \frac{1}{4}\pi$
7. (a) The sequence diverges because the terms indexed by odd numbers converges to $\frac{1}{2}$ and the terms indexed by even numbers converges to $-\frac{1}{2}$. (b) The series diverges by the vanishing criterion (*i.e.*, $\because\ a_n \not\to 0$).
8. (a) The series diverges by the ratio test (the ratio of successive terms diverges to ∞). (b) The series converges by the integral test, or by the comparison (or limit comparison) test with *e.g.*, $b_n = n^{-3/2}$. (c) The series converges by the root test ($\sqrt[n]{|a_n|} \to \frac{3}{5}$). (d) The series converges by the comparison (or limit comparison) test with $b_n = n^{-3}$.
9. (a) The series is conditionally convergent. It converges by the alternating series test because $(\ln n)^{-1} \searrow 0$ as $n \to \infty$, but does not converge absolutely by the comparison test with, *e.g.*, $b_n = n^{-1}$. (b) The series is absolutely convergent by the ratio test or the root test. $|a_{n+1}/a_n|$ and $\sqrt[n]{|a_n|}$ both converge to $\frac{1}{2}$.
10. The radius of convergence of this series is 2, and its interval of convergence is $(-1, 3)$. The series diverges at the endpoints of its interval of convergence by the vanishing criterion.
11. (a) The Maclaurin series of $\cos x$ begins

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

and (b) the entire series, in summation notation, is

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}.$$

12. (a) $-\frac{6}{7}$, (b) 1.