CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB)

1. Find $\dfrac{dy}{dx}$ for

   (a) $y = \ln\left(\arctan\left(x^3\right)\right)$ (b) $y = x\arcsin(2x) + \operatorname{arcsec}(e^x)$

2. Evaluate the following integrals.

   (a) $\displaystyle\int \sin^5(2x)\cos^3(2x)\,dx$ (b) $\displaystyle\int_1^5 \frac{dx}{x(x+2)}$

   (c) $\displaystyle\int x\arctan x\,dx$ (d) $\displaystyle\int \frac{x^3\,dx}{\sqrt{4x^2-9}}$ (e) $\displaystyle\int \frac{e^x\,dx}{7+e^{2x}}$

   (f) $\displaystyle\int_0^{3/\sqrt{2}} \frac{dx}{\sqrt{9-x^2}}$ (g) $\displaystyle\int \frac{x+1}{(x-1)(x+4)^2}\,dx$

3. Calculate the following limits.

   (a) $\displaystyle\lim_{x\to 1} \frac{x-e^{x-1}}{(x-1)^2}$ (b) $\displaystyle\lim_{x\to\infty}\left(\frac{x-1}{x+2}\right)^x$

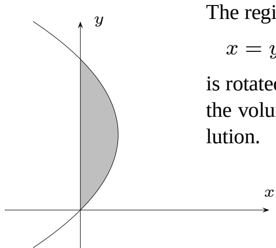
4. Evaluate or show divergence.

   (a) $\displaystyle\int_{\frac{1}{2}}^{\infty}\left(\frac{1}{x}-\frac{2}{2x+1}\right)dx$ (b) $\displaystyle\int_{-1}^{2}\frac{x+1}{(x^2+2x)^{4/3}}\,dx$

5. (a) The region bounded by

   $x = y - y^2$ and $x = 0$

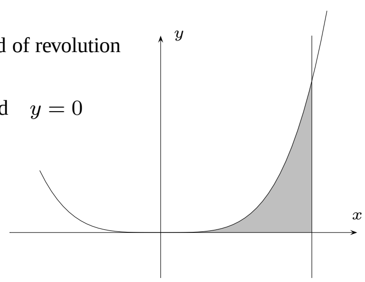
   is rotated about the y-axis. Find the volume of the solid of revolution.



   (b) Find the volume of the solid of revolution if the region bounded by

   $y = x^4$, $x = 1$ and $y = 0$

   is rotated about the y-axis.



6. Give the particular solution for the following differential equation.

$$(1+x^2)y' = 2x\sqrt{1-y^2}, \quad y(0) = \frac{1}{2}$$

7. Given the following series,

$$\sum_{n=1}^{\infty}\frac{4}{(4n-1)(4n+3)},$$

   let $\{s_n\}$ be the sequence of partial sums.
   (a) Find s_1, s_2, s_3 and s_n.
   (b) Find the sum of the series.

8. Test these sequences for convergence. If a sequence converges find its limit.

   (a) $\left\{\dfrac{2n}{2n^2-1}\right\}$ (b) $\left\{\dfrac{\cos n}{\ln n}\right\}$ (c) $\left\{\dfrac{e^n}{n}\right\}$

9. Determine by using an appropriate test whether these series converge.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{3^{n+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{n}-\frac{1}{n^2}\right)$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{2n+3}}{5n^2-2}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n}$

10. Are the following series divergent, absolutely convergent, or conditionally convergent? State which tests are used in each case.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{3n+2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(n!)^2}{(2n)!}$

11. Find the interval of convergence of

$$\sum_{n=1}^{\infty}\frac{3^n(x-2)^n}{n+1}.$$

12. Find the first five non-zero terms of the Maclaurin series of

$$f(x) = \sqrt{3x+4}.$$

ANSWERS

1. (a) $\dfrac{3x^2}{(1+x^6)\arctan x^3}$, (b) $\arcsin 2x + \dfrac{2x}{\sqrt{1-4x^2}} + \dfrac{1}{\sqrt{e^{2x}-1}}$.

2. (a) $\frac{1}{12}\sin^6 2x - \frac{1}{16}\sin^8 2x + C$, (b) $\frac{1}{2}\ln\frac{15}{7}$,
   (c) $\frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + C$, (d) $\frac{1}{24}(2x^2+9)\sqrt{4x^2-9}+C$,
   (e) $\frac{1}{\sqrt{7}}\arctan\dfrac{e^x}{\sqrt{7}} + C$, (f) $\frac{\pi}{4}$, (g) $\frac{2}{25}\ln\left|\dfrac{x-1}{x+4}\right| - \dfrac{3}{5(x+4)} + C$

3. (a) $-\frac{1}{2}$, (b) e^{-3}

4. (a) The integral converges to $\ln 2$. (b) The integral diverges ($p = \frac{4}{3} > 1$).

5. (a) $V = \pi\displaystyle\int_0^1 (y-y^2)\,dy = \frac{\pi}{30}$ cubic units.

   (b) $V = 2\pi\displaystyle\int_0^1 x\cdot x^4\,dx = \frac{\pi}{3}$ cubic units.

6. $y = \sin\left(\ln(1+x^2)+\frac{\pi}{6}\right)$

7. (a) $s_n = \frac{1}{3} - \frac{1}{4n+3}$, $\therefore$ (b) the sum of the series is $\lim s_n = \frac{1}{3}$.

8. NB: The general term of the sequence in question will be denoted by a_n. (a) $a_n = (2/n)/(2-1/n^2) \to 0$. (b) $|a_n| \leqslant 1/\ln n \to 0$, $\therefore a_n \to 0$. (c) $\lim e^n/n \overset{\ell\text{'HR}}{=} \lim e^n = \infty$, so $\{a_n\}$ diverges.

9. NB: The general term of the series in question will be denoted by a_n. (a) This is a *convergent* geometric series ($r = 2/3$). (b) $a_n \geqslant 1/n$, $\therefore$ $\sum a_n$ diverges. (c) $a_n/n^{-3/2} \to \frac{1}{5}\sqrt{2}$, $\therefore$ $\sum a_n$ converges by the limit comparison test. (d) $a_n > 1/n$ if $n \geqslant 3$, $\therefore$ $\sum a_n$ diverges.

10. NB: The general term of the series in question will be denoted by $(-1)^n a_n$. (a) $a_n \downarrow 0$, so $\sum(-1)^n a_n$ converges by the alternating series test; however, $a_n/n^{-1} \to \frac{1}{3}$, so $\sum a_n$ diverges by the limit comparison test. $\therefore$ $\sum(-1)^n a_n$ is conditionally convergent. (b) $a_{n+1}/a_n = \frac{1}{2}(n+1)/(2n+1) \to \frac{1}{4}$, $\therefore$ $\sum(-1)^n a_n$ is absolutely convergent.

11. $\left[\frac{5}{3}, \frac{7}{3}\right)$

12. $$\sqrt{3x+4} = 2 + \sum_{k=1}^{\infty}\frac{(-1)^{k-1}3^k(2k)!}{2^{4k-1}(k!)^2(2k-1)}\,x^k$$
$$= 2 + \frac{3}{4}x - \frac{9}{64}x + \frac{27}{512}x^3 - \frac{405}{16384}x^4$$