# CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/$\infty$)

1. Compute the derivative of $y = \arctan\{(1+x)^{-1}\}$ and simplify your answer.

2. Evaluate the following integrals.

   (a) $\int \cos^4 2x \, dx$ (b) $\int_2^4 \frac{(\operatorname{arcsec}\sqrt{x})^3}{x\sqrt{x-1}} \, dx$

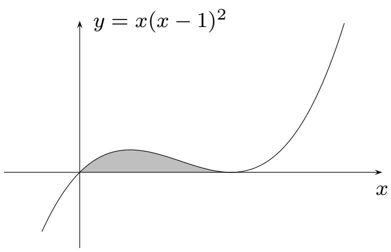
   (c) $\int x(\ln 5x)^2 \, dx$ (d) $\int \frac{x^2 \, dx}{\sqrt{x-4}}$

   (e) $\int \sec^4 3x \tan^4 3x \, dx$ (f) $\int \frac{8x^2+4x+5}{(x+1)^2(2x-1)} \, dx$

   (g) $\int_0^{\frac{1}{2}} x \arcsin x \, dx$ (h) $\int \frac{dx}{\sqrt{4x^2-9}}$

3. Calculate the area of the region bounded by the graphs of $y = \ln x$, $y = 1$, $x = e^2$.

4. (a) Calculate the volume obtained by rotating the region bounded by $y = 0$ and $y = x(x-1)^2$ about the $y$-axis.




   (b) Set up, but do not evaluate, the integral representing the volume obtained by rotating the same region about the $x$-axis.

5. Evaluate the following limits.

   (a) $\lim_{x\to 0^+} \left( \frac{1}{\sin x} - \frac{1}{x} \right)$ (b) $\lim_{x\to 0^+} (1 + 2\tan x)^{1/x}$

6. Determine whether each integral converges. If an integral converges, give its value.

   (a) $\int_{-\infty}^4 \frac{dx}{\sqrt[5]{(4-x)^2}}$ (b) $\int_0^3 \frac{dx}{9-x^2}$

7. Solve the differential equation

$$\cos^2 x \frac{dy}{dx} = e^{-y} \sin x; \quad y(0) = 0.$$

8. Let $\{s_n\}$ denote the sequence of partial sums of the series

$$\sum_{n=1}^{\infty} \frac{9}{(3n-1)(3n+2)}.$$

   (a) Give a general formula for $s_n$.

   (b) Find the sum of the series.

9. Determine whether each series converges or diverges. State the test you are using and display a proper solution.

   (a) $\sum_{n=1}^{\infty} \frac{\pi + 3\cos n}{n}$ (b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{4n^2-1}}$ (c) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

10. Determine whether each series is absolutely or conditionally convergent, or divergent. Justify your answers completely.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+3\sqrt{n}}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}}{n!}$ (c) $\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^{n^2}$

11. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+4)^n}{5^n (2n+1)}.$$

12. (a) Find the first four nonzero terms of the Maclaurin series of $\ln(1+x)$.

   (b) Express the Maclaurin series of $\ln(1+x)$ using sigma-notaton.

## ANSWERS

1. $-(x^2+2x+2)^{-1}$

2. (a) $\frac{3}{8}x + \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$

   (b) $\frac{175}{41472}\pi^4$

   (c) $\frac{1}{4}x^2\{2(\ln 5x)^2 - 2\ln 5x + 1\} + C$

   (d) $\frac{2}{15}(3x^2+16x+128)\sqrt{x-4} + C$

   (e) $\frac{1}{21}\tan^7 3x + \frac{1}{15}\tan^5 3x + C$

   (f) $2\ln|2x^2+x-1| + 3(x+1)^{-1} + C$

   (g) $\frac{1}{48}(3\sqrt{3} - \pi)$

   (h) $\frac{1}{2}\ln|2x + \sqrt{4x^2-9}\,| + C$

3. $\int_e^{e^2} (\ln x - 1) \, dx = e$

4. (a) $2\pi\int_0^1 x^2(x-1)^2 \, dx = \frac{2}{15}\pi$ (b) $\pi\int_0^1 x^2(x-1)^4 \, dx = \frac{1}{105}\pi$

5. (a) 0 (b) $e^2$

6. (a) The integral diverges. (b) The integral diverges.

7. $y = \ln\sec x$

8. (a) $s_n = \frac{3}{2} - 3(3n+2)^{-1}$ (b) $\frac{3}{2}$

9. (a) $(\pi - 3)n^{-1} < a_n$ for $n \geqslant 1$, so $\sum a_n$ diverges by the comparison text.

   (b) $\lim a_n = \frac{1}{2}$, so $\sum a_n$ diverges by the vanishing criterion.

   (c) $a_n < n^{-2}$ for $n \geqslant 1$, so $\sum a_n$ converges by the comparison test.

10. (a) The series is conditionally convergent: $|a_n| \downarrow 0$, so $\sum a_n$ converges by the alternating series test; $|a_n| \geqslant \frac{1}{5}n^{-1}$ for $n \geqslant 1$, so $\sum |a_n|$ diverges by the comparison test.

   (b) $|a_{n+1}/a_n| = 9/(n+1) \to 0$, so $\sum a_n$ is absolutely convergent by the ratio test.

   (c) $\sqrt[n]{|a_n|} = \left(1 + \frac{1}{n}\right)^n \to e$, so $\sum a_n$ is divergent by the root test.

11. The radius of convergence of the given series is 5 and its interval of convergence is $(-9, 1]$.

12. $\underbrace{x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots}_{(a)} = \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}}_{(b)}$