1. Differentiate

$$y = \sqrt{x}\arcsin\sqrt{x} + \sqrt{1-x}$$

with respect to x and simplify your answer.

2. Evaluate the integrals.

(a) $\displaystyle\int_0^1 \frac{e^{\arctan x}}{1+x^2}\,dx$ (b) $\displaystyle\int \frac{x\,dx}{\sqrt{2x-1}}$

(c) $\displaystyle\int e^{5x}\cos x\,dx$ (d) $\displaystyle\int \sin^5 3x\,dx$

(e) $\displaystyle\int \tan^3\left(\tfrac{1}{2}x\right)\sec^3\left(\tfrac{1}{2}x\right)dx$ (f) $\displaystyle\int \frac{x^2\,dx}{\sqrt{9-x^2}}$

(g) $\displaystyle\int \frac{x^2-4x+5}{(x+1)(x^2+4)}\,dx$

3. Evaluate the improper integrals.

(a) $\displaystyle\int_0^\infty xe^{-x^2}\,dx$ (b) $\displaystyle\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$

4. Evaluate the limits.

(a) $\displaystyle\lim_{x\to\infty}(\ln x)^{e^{-x}}$ (b) $\displaystyle\lim_{x\to 0}\frac{x - x\cos x}{\sin x - x}$

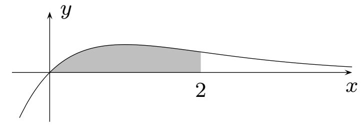
(c) $\displaystyle\lim_{x\to\infty}\left\{\ln(x+1) - \ln(2x+3)\right\}$

5. Compute the area of the region bounded by the graphs of

$$y = 4x - x^2 \quad\text{and}\quad y = 8 - 2x.$$

6. Let $\mathscr{R}$ be the shaded region in the figure, which is bounded by the graphs of

$$y = xe^{-x},\quad y = 0,\quad x = 0,\quad\text{and}\quad x = 2.$$



(a) Set up the integrals required to compute the volume of the solid obtained by rotating $\mathscr{R}$ about: (i) the x-axis, (ii) the y-axis.
(b) Evaluate *one* of the integrals from part (a).

7. Solve the differential equation

$$yy' = (x+3)(y^2+3);\quad y(-6) = -2.$$

8. Determine whether each series converges or diverges; if it converges, find the sum. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}\left\{\arctan n - \arctan(n+1)\right\}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^{2n}}{3^{n+1}}$

9. Determine whether each series converges or diverges. State the tests you use and verify that the conditions for using them are satisfied.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n+2}{2n+1}\right)^n$ (b) $\displaystyle\sum_{k=1}^{\infty}\frac{e^k}{k}$ (c) $\displaystyle\sum_{k=1}^{\infty}\frac{\operatorname{arcsec} k}{k}$

10. Label each series as absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$ (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{5^n}{(n+1)!}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n+1}}{n^2}$

11. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-1)^n}{n2^n}.$$

12. Find the Taylor series of $f(x) = \ln x$ centred at 1.

ANSWERS

1. $(\arcsin\sqrt{x})/(2\sqrt{x})$
2. (a) $e^{\pi/4} - 1$
   (b) $\frac{1}{3}(x+1)\sqrt{2x-1} + C$
   (c) $\frac{1}{26}e^{5x}(\sin x + 5\cos x) + C$
   (d) $-\frac{1}{15}\cos^5 3x + \frac{2}{9}\cos^3 3x - \frac{1}{3}\cos 3x + C$
   (e) $\frac{2}{5}\sec^5\left(\frac{1}{2}x\right) - \frac{2}{3}\sec^3\left(\frac{1}{2}x\right) + C$
   (f) $\frac{9}{2}\arcsin\left(\frac{1}{3}x\right) - \frac{1}{2}x\sqrt{9-x^2} + C$
   (g) $\ln\dfrac{(x+1)^2}{\sqrt{x^2+4}} - \frac{3}{2}\arctan\left(\frac{1}{2}x\right) + C$
3. (a) $\frac{1}{2}$ (b) $\frac{1}{3}\pi$
4. (a) 1 (b) -3 (c) $-\ln 2$
5. $\frac{4}{3}$.
6. (i) $V = \pi\displaystyle\int_0^2 x^2e^{-2x}\,dx = \frac{1}{4}\pi\left(1 - 13e^{-4}\right)$
   (ii) $V = 2\pi\displaystyle\int_0^2 x^2e^{-x}\,dx = 4\pi\left(1 - 5e^{-2}\right)$
7. $y = -\sqrt{7e^{(x+3)^2-9} - 3}$
8. (a) The sum of the series is $-\frac{1}{4}\pi$.
   (b) The series diverges because it is a geometric series with $|r| = \left|-\frac{4}{3}\right| \geqslant 1$.
9. (a) The series converges by the root test.
   (b) The series diverges by the vanishing criterion.
   (c) The series diverges with the harmonic series by the comparison test.
10. (a) The series is conditionally convergent (by the alternating series test, and because the associated series of non-negative terms is a logarithmic p-series with $p \leqslant 1$).
    (b) The series is absolutely convergent by the ratio test.
    (c) The series is absolutely convergent by the comparison test (with a convergent p-series).
11. $(-1, 3]$
12. $\ln x = \ln\left\{1 + (x-1)\right\} = \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-1)^n}{n}$.