Hereafter, according to the context, I denotes the integral in question, ℓ the limit in question, A the area in question, V the volume in question, and a_n (or a_k) the sequence of terms of the series in question.

1. $\dfrac{dy}{dx} = \dfrac{\arcsin\sqrt{x}}{2\sqrt{x}} + \dfrac{\sqrt{x}}{2\sqrt{x}\sqrt{1-x}} - \dfrac{1}{2\sqrt{1-x}} = \dfrac{\arcsin\sqrt{x}}{2\sqrt{x}}$

2. (a) $I = \displaystyle\int_0^{\frac14\pi} e^t\,dt = e^{\pi/4} - 1 \quad [t = \arctan x]$

   (b) $I = x\sqrt{2x-1} - \displaystyle\int \sqrt{2x-1}\,dx = \tfrac13(x+1)\sqrt{2x-1} + C$

   (c)

$+$	e^{5x}	$\cos x$
$-$	$5e^{5x}$	$\sin x$
$+$	$25e^{5x}$	$-\cos x$

   $I = e^{5x}(\sin x + 5\cos x) - 25I$

   $\therefore\quad I = \tfrac{1}{26}e^{5x}(\sin x + 5\cos x) + C$

   (d) $I = -\tfrac13\displaystyle\int (t^2-1)^2\,dt = -\tfrac{1}{15}\cos^5 3x + \tfrac29\cos^3 3x - \tfrac13\cos 3x + C$ $\quad [t = \cos 3x]$

   (e) $I = -2\displaystyle\int t^{-6}(1-t^2)\,dt = \tfrac25\sec^5\big(\tfrac12 x\big) - \tfrac23\sec^3\big(\tfrac12 x\big) + C$ $\quad [t = \cos\tfrac12 x]$

   (f) $I = 9\displaystyle\int \sin^2\vartheta\,d\vartheta = \tfrac92\arcsin\big(\tfrac13 x\big) - \tfrac12 x\sqrt{9-x^2} + C \quad [x = 3\sin\vartheta]$

   (g) $I = \displaystyle\int \left\{\frac{2}{x+1} - \frac{x+3}{x^2+4}\right\} dx$

   $= 2\ln|x+1| - \tfrac12\ln(x^2+4) - \tfrac32\arctan\big(\tfrac12 x\big) + C$

3. (a) $I = \displaystyle\lim_{t\to\infty}\left\{-\tfrac12 e^{-x^2}\right\}\Big|_0^t = \tfrac12$ (b) $I = \displaystyle\lim_{t\to1^+}(\operatorname{arcsec} x)\Big|_t^2 = \tfrac13\pi$

4. (a) $\displaystyle\lim_{x\to\infty}\frac{\ln\ln x}{e^x} = \lim_{x\to\infty}\frac{1}{e^x\,x\ln x} = 0, \quad \therefore \lim_{x\to\infty}(\ln x)^{e^{-x}} = 1.$

   (b) $\ell = \displaystyle\lim_{x\to0}\frac{1-\cos x + x\sin x}{\cos x - 1} = \lim_{x\to0}\frac{\frac{1-\cos x}{x^2} + \frac{\sin x}{x}}{\frac{\cos x - 1}{x^2}}$

   $= \dfrac{\frac12 + 1}{-\frac12} = -3$

   (c) $\ell = \displaystyle\lim_{x\to\infty}\ln\frac{x+1}{2x+3} = -\ln 2$

5. The curves meet where $x = 2, 4$, & $A = \displaystyle\int_2^4 \{(4x-x^2)-(8-2x)\}\,dx = \tfrac43$.

   Alternatively, a sketch reveals that $A = 4 - \displaystyle\int_0^2 x^2\,dx = \tfrac43$.

6. (i) $V = \pi\displaystyle\int_0^2 x^2 e^{-2x}\,dx$ .... (a)

   $= \left\{-\tfrac14\pi e^{-2x}(2x^2+2x+1)\right\}\Big|_0^2 = \tfrac14\pi\big(1 - 13e^{-4}\big)$ .... (b)

   (ii) $V = 2\pi\displaystyle\int_0^2 x^2 e^{-x}\,dx$ .... (a)

   $= \left\{-2\pi e^{-x}(x^2+2x+2)\right\}\Big|_0^2 = 4\pi\big(1 - 5e^{-2}\big)$ .... (b)

7. $\displaystyle\int \frac{y\,dy}{y^2+3} = \int (x+3)\,dx \implies \ln(y^2+3) = (x+3)^2 + C$

   $\implies y^2 + 3 = e^{(x+3)^2 + C}$

   $\implies y = -\sqrt{7e^{(x+3)^2-9} - 3}$, using $y(-6) = -2$.

8. (a) The sum of the series is $\displaystyle\lim_{n\to\infty}\left\{\tfrac14\pi - \arctan(n+1)\right\} = -\tfrac14\pi$.

   (b) The series diverges because it is a geometric series with $|r| = \left|-\tfrac43\right| \geqslant 1$.

9. (a) $\sqrt[n]{|a_n|} = \frac{n+2}{2n+1} \to \tfrac12 < 1, \ \therefore \sum a_n$ converges by the root test.

   (b) $a_k \to \infty, \ \therefore \sum a_k$ diverges by the vanishing criterion.

   (c) $a_k \geqslant \tfrac13\pi k^{-1}$ if $k \geqslant 2, \ \therefore \sum a_k$ diverges with the harmonic series by the comparison test.

10. (a) $\sum a_n$ converges by the alternating series test because $|a_n| \downarrow 0$, but not absolutely, *e.g.*, because $\sum|a_n|$ is a logarithmic p-series with $p \leqslant 1$, or by the Cauchy condensation test, or by the integral test; $\therefore \sum a_n$ is conditionally convergent.

    (b) $|a_{n+1}/a_n| = 5(n+2)^{-1} \to 0 < 1, \ \therefore \sum a_n$ is absolutely convergent by the ratio test.

    (c) $|a_n| < n^{-3/2}, \ \therefore \sum a_n$ is absolutely convergent by comparison with a convergent p-series.

11. $|a_{n+1}/a_n| \to \tfrac12|x-1| < 1 \iff -1 < x < 3$. If $x = -1$ the series is $\sum\big(-n^{-1}\big)$, which diverges with the harmonic series, and if $x = 3$ the series is $\sum\big\{(-1)^{n-1}n^{-1}\big\}$, which converges by the alternating series test (or, because it is the alternating harmonic series). Therefore, the interval of convergence of the series is $(-1, 3]$.

12. $\ln x = \ln\{1 + (x-1)\} = \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-1)^n}{n}$.