

1. Evaluate the following integrals.

(a) $\int x^2 e^{-x^3} dx$

(b) $\int_1^{e^2} \frac{dx}{x(4 + \ln x)}$

(c) $\int x^2 \sqrt{2x-1} dx$

(d) $\int x \ln(x+1) dx$

(e) $\int \tan^5(5x) \sec^4(5x) dx$

(f) $\int \frac{x+2}{\sqrt{x^2+4}} dx$

(g) $\int \frac{dx}{(2x-1)(1-3x)}$

(h) $\int_0^1 \frac{dx}{\sqrt{3+2x-x^2}}$

2. Calculate the following limits.

(a) $\lim_{x \rightarrow 0} \frac{x - \arctan x}{x}$

(b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^x$

3. Given: $f(x) = xe^{-x}$.

(a) Show that $\int_1^{\infty} f(x) dx$ is convergent.

(b) Show that $f(x)$ is decreasing on $(1, \infty)$.

(c) Does the series $\sum_{k=1}^{\infty} ke^{-k}$ converge? *Justify your answer.*

4. Determine the convergence or divergence for $\int_{-1}^1 \frac{3}{x} dx$.

5. Find the volume of the solid generated when the region bounded by the graphs of $y = x^2$ and $y = x^3$ is revolved about the y -axis.

6. Set up a definite integral which expresses the volume of the solid generated when the region bounded by the graphs of $y = x^2 - 6x + 9$ and $y = 9$ is revolved about the x -axis. *Do not evaluate the integral.*

7. Given the sequence $\left\{ \frac{n}{\ln(n+1)} \right\}_{n=1}^{\infty}$

(a) List the first four terms of the sequence.

(b) Determine whether the sequence converges, and if so, find its limit.

8. Consider the sequence $\{a_n\} = \left\{ \frac{n-1}{n+1} \right\}_{n=1}^{\infty}$.

(a) Show that $\{a_n\}$ is monotonic.

(b) Is $\{a_n\}$ convergent? *Justify your answer.*

9. Consider the series $\sum_{k=1}^{\infty} \left(\frac{1}{k+1} - \frac{1}{k+2} \right)$.

(a) Find s_n , the sum of the first n terms of the series.

(b) Find $\lim_{n \rightarrow \infty} s_n$.

(c) Does the series converge? If it converges, find its sum.

10. Determine if the following series converge or diverge. State which test you are using and give sufficient details to show that you are using the test correctly.

(a) $\sum_{k=1}^{\infty} \frac{k^2 - 1}{5k^2 + 7}$ (b) $\sum_{k=1}^{\infty} \frac{5^k}{(k+1)!}$ (c) $\sum_{k=1}^{\infty} \frac{\sin^2 k}{k^2}$

(d) $\sum_{k=1}^{\infty} \frac{k-1}{k^2+k}$ (e) $\sum_{k=1}^{\infty} \frac{4^k}{3^{k+1}}$

11. Determine whether the following series converge absolutely, converge conditionally, or diverge. *Justify your answers.*

(a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k+2}}$ (b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^k}$

12. Find the radius and interval of convergence for $\sum_{k=1}^{\infty} \frac{(x-4)^k}{k5^k}$.

13. Find the first *four* terms of the Taylor polynomial for $f(x) = \frac{1}{x}$ about $x = 1$.