

1. Evaluate each of the following integrals.

a. $\int (x+2)\sqrt{x-3} dx$

b. $\int_1^e \frac{(\ln x)^2}{x^4} dx$

c. $\int_{\sqrt{2}}^2 \frac{dx}{x^2\sqrt{x^2-1}}$

d. $\int \frac{3x^2}{\sqrt{4-x^2}} dx$

e. $\int \sin^4 x \cos^3 x dx$

f. $\int \arcsin x dx$

g. $\int \frac{dx}{x^3+2x^2+x}$

h. $\int \frac{x^2-2x+3}{x^3-1} dx$

2. Find the limits

a. $\lim_{x \rightarrow 0^+} \tan x \ln x$

b. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{\cos 2x - 1}$

c. $\lim_{x \rightarrow 0^+} (e^{-2x} + x)^{1/x}$

3. Evaluate the integral, or show that it diverges

a. $\int_0^\infty \frac{e^{-2\sqrt{x}} \cos \sqrt{x}}{\sqrt{x}} dx$

b. $\int_0^\pi \sqrt{\frac{1+\sin \vartheta}{1-\sin \vartheta}} d\vartheta$

4. Refer to the graph below for this question.

a. Find the sum of areas of the regions \mathcal{R}_1 and \mathcal{R}_2 .

b. Find the volume of the solid generated when region \mathcal{R}_2 is revolved about

- i. the x -axis,
- ii. the line $x = 3$.

5. Find the limit of the sequence $\left\{ \frac{\ln(2+e^n)}{3n} \right\}$.

6. Find each of the following series. Write out the first four terms explicitly, and indicate the general term using summation notation.

- a. The Taylor series of $f(x) = x^{2/3}$, centred at 1.
- b. The Maclaurin series of $f(x) = \ln(3-2x)$.

7. Determine if the following series converge or diverge. State which test you are using and give sufficient details to show that you are using the test correctly.

a. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2}$

b. $\sum_{k=1}^{\infty} \frac{k!}{e^{k^2}}$

c. $\sum_{k=0}^{\infty} \frac{2k-1}{3k+5}$

d. $\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n^2 + 1}}$

8. Classify each of the following series as absolutely convergent, conditionally convergent, or divergent. Support your conclusions with sufficient details.

a. $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln k \sqrt[3]{k+1}}$

b. $\sum_{n=0}^{\infty} (-1)^n \frac{n^2+2}{n^4+3}$

9. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(5x+3)^n}{2^n \sqrt{n+4}}.$$

