CEGEP
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COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201-NYB)

1. Given $y = \arcsin(2x^2 - 1)$, find $\frac{dy}{dx}$ and simplify your answer.

2. Let $\mathscr{R}$ be the region bounded by the graphs of
$$y = x, \quad x = 0, \quad \text{and} \quad y^2 = x + 2.$$
   a. Find the area of $\mathscr{R}$.
   b. Find the volume of the solid of revolution obtained when $\mathscr{R}$ is revolved about i. the x-axis, and ii. the line $x = 2$.

3. Evaluate the following integrals.
   a. $\displaystyle\int x^2 \sin 2x \, dx$ b. $\displaystyle\int \sec^4 x \tan^4 x \, dx$ c. $\displaystyle\int_0^4 \frac{3x \, dx}{\sqrt{x^2+9}}$
   d. $\displaystyle\int \frac{3x - 18}{(x^2+4)(x-1)(x+2)} \, dx$ e. $\displaystyle\int \frac{dx}{x\{(\ln x)^2 + 9\}}$
   f. $\displaystyle\int \frac{x^2 \, dx}{\sqrt{4 - x^2}}$ g. $\displaystyle\int e^{2x} \sin x \, dx$

4. Evaluate the following limits.
   a. $\displaystyle\lim_{x\to\infty} \frac{(\ln x)^2}{x}$ b. $\displaystyle\lim_{x\to\infty} \left(1 + \frac{5}{3x}\right)^{x/2}$ c. $\displaystyle\lim_{x\to 0+} \left\{\frac{1}{e^x - 1} - \frac{1}{x}\right\}$

5. Evaluate the integral or explain why it does not converge.
   a. $\displaystyle\int_5^\infty \frac{dx}{x^3\sqrt{x^2 - 25}}$ b. $\displaystyle\int_{-\infty}^0 \frac{e^x \, dx}{3 - 2e^x}$

6. Solve the differential equation
$$y^2 - 2\sqrt{x^2+1}\,\frac{dy}{dx} = 1; \quad y(0) = 2.$$
   Express y as an explicit function of x, in simplified form.

7. Consider the sequence $\{a_n\}$ defined by
$$a_0 = \tfrac{1}{2}, \quad \text{and} \quad a_{n+1} = 2a_n - a_n^2\sqrt{2} \text{ if } n \geqslant 1.$$
   a. Show that $\frac{1}{2} < a_n < \frac{1}{2}\sqrt{2}$ if $n \geqslant 1$.
   b. Show that $\{a_n\}$ is increasing.
   c. Is the sequence $\{a_n\}$ convergent? If so, explain why and determine $\lim a_n$. If not, explain why not.
   d. Is the series $\sum a_n$ convergent? Justify your answer completely.

8. Find the sum of each series or explain why it diverges.
   a. $\displaystyle\sum_{n=1}^\infty \left(\frac{1}{n+3} - \frac{1}{n+4}\right)$ b. $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \dots$

9. Determine whether each series converges. Justify your answers completely.
   a. $\displaystyle\sum_{n=1}^\infty \frac{5^n}{n^8}$ b. $\displaystyle\sum_{n=1}^\infty \sqrt{n}\tan\left(\sqrt[3]{n^{-5}}\right)$
   c. $\displaystyle\sum_{n=2}^\infty \frac{1}{n \ln n}$ d. $\displaystyle\sum_{n=1}^\infty \frac{4n^3 + 2n - 1}{3n^5 - n^3 + 2}$

10. Determine whether each series converges absolutely, converges conditionally, or diverges. Justify your answers completely.
   a. $\displaystyle\sum_{n=1}^\infty \frac{(-1)^{n+1}}{\ln(n+1)}$ b. $\displaystyle\sum_{n=1}^\infty \frac{\cos \pi n}{\sqrt[n]{(\ln n)^{n \ln n - 1}}}$

11. Find the interval of convergence of the series
$$\sum_{n=0}^\infty \frac{(-2)^n \cdot 2 \cdot 5 \cdots (3n-1)}{15^n n!}(x+2)^n.$$

12. Find the Taylor series of $f(x) = \sqrt{x+1}$ centred at 2.