CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/∞)

1. Differentiate: $f(x) = \operatorname{arcsec}\left(x^2 - 1\right)$.

2. Evaluate the following integrals:

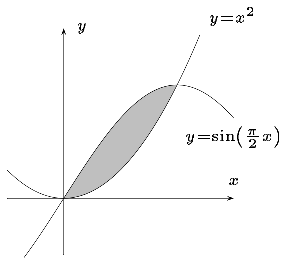
   (a) $\displaystyle\int \frac{dx}{(4+5x)^{3/2}}$ (b) $\displaystyle\int \frac{1+\cos\vartheta+\sin\vartheta\cos\vartheta}{\cos^2\vartheta}\,d\vartheta$

   (c) $\displaystyle\int_1^{\sqrt{3}} x^3\sqrt{1+x^2}\,dx$ (d) $\displaystyle\int \frac{2+x}{\sqrt{9-x^2}}\,dx$

   (e) $\displaystyle\int_0^1 \arctan x\,dx$ (f) $\displaystyle\int \tan^3(2x)\;dx$

   (g) $\displaystyle\int_3^6 \frac{\sqrt{x^2-9}}{x}\,dx$ (h) $\displaystyle\int \frac{3x^2+4x+12}{x\,(x+1)^2}\,dx$

3. Consider the shaded region bounded by $y = \sin\left(\frac{\pi}{2}x\right)$ and $y = x^2$, between $x = 0$ and $x = 1$.



   (a) Find the area of this region.
   (b) FInd the volume of the solid generated by revolving this region about the x-axis.
   (c) Set up the integral, *but do not evaluate it,* for the volume of the solid generated by revolving the region about the y-axis.

4. Evaluate the following limits:

   (a) $\displaystyle\lim_{x\to 0^+} \arcsin(x)\csc(\pi x)$ (b) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{\cos x}{x^2}\right)$

   (c) $\displaystyle\lim_{x\to\infty}(e^x + x)^{2/x}$

5. Determine divergence or convergence (and evaluate if convergent) for the following integrals:

   (a) $\displaystyle\int_0^\infty xe^{-x^2}\,dx$ (b) $\displaystyle\int_{-1}^1 \frac{1}{x^2}\,dx$

6. Consider the sequence defined by $a_1 = 3$ and $a_n = \frac{1}{5}a_{n-1}$.

   (a) Write the first four terms: $a_1,\ a_2,\ a_3,\ a_4$.
   (b) Write an expression for the n$^{\text{th}}$ term of this sequence that is *not* a recursive definitinon.
   (c) Compute $\displaystyle\lim_{n\to\infty} a_n$.

7. Determine whether the geometric series
$$1 - \frac{5}{8} + \frac{25}{64} - \frac{125}{512} + \frac{625}{1024} - \cdots$$
converges or diverges. If the series converges, find its sum.

8. Determine whether the following series converge or diverge. State which test you are using, and show that any conditions for the application of the test have been met.

   (a) $\displaystyle\sum_{n=2}^\infty \frac{(-1)^n}{\ln n}$ (b) $\displaystyle\sum_{n=1}^\infty \frac{|\sin n|}{n^2+1}$

   (c) $\displaystyle\sum_{n=1}^\infty \frac{e^{-\sqrt{n}}}{\sqrt{n}}$ (d) $\displaystyle\sum_{n=1}^\infty \frac{3n^2-5}{4n^2+7}$

9. Do the following series converge absolutely, converge conditionally, or diverge? Justify your conclusions.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n\,n!}{n3^n}$ (b) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^{n+1}}{\sqrt{2n+1}}$

10. Determine the interval of convergence for the power series
$$\sum_{n=1}^\infty \frac{(-1)^{n+1}(x-1)^n}{n}.$$

11. Use the Maclaurin series:
$$\cos x = \sum_{n=1}^\infty (-1)^n\frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

   (a) to find the Maclaurin series for $\cos\left(x^{3/2}\right)$,

   (b) to approximate $\displaystyle\int_0^1 \cos(x^{3/2})\,dx$ with an error smaller than 0.001.