CEGEP
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COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/∞)

1. Differentiate

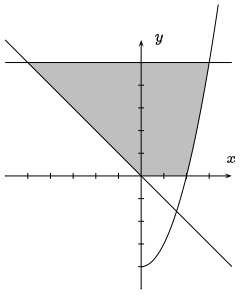
   (a) $y = e^{3x} \arccos \sqrt{x}$,

   (b) $y = (\arctan x)^2$.

2. Let R be the region bounded by

   $$x = \sqrt{y+4}, \qquad y = -x,$$
   $$y = 0, \quad \text{and} \quad y = 5.$$

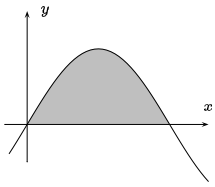
   Find the area of R.



3. Let R be the region bounded by one half-cycld of $y = \sin x$, between 0 and π, and the x-axis.

   (a) Find the volume of the solid of revolution obtained by revolving R about the x-axis.

   (b) Set up the integral required to find the volume of the solid of revolution obtained by revolving R about the y-axis. *Do not evaluate the integral.*



4. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{\ln x}{x}\,dx$

   (b) $\displaystyle\int \frac{\sqrt{4x^2-9}}{x}\,dx$

   (c) $\displaystyle\int \sec^4\left(\frac{x}{2}\right)\tan^2\left(\frac{x}{2}\right)\,dx$

   (d) $\displaystyle\int \frac{4x-3}{x^2+16}\,dx$

   (e) $\displaystyle\int \frac{6x^2-16x+3}{(x-2)^2(2x+1)}\,dx$

   (f) $\displaystyle\int_0^1 x \arctan x\,dx$

5. Calculate the following limits.

   (a) $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

   (b) $\displaystyle\lim_{x\to\infty}\left(\frac{2x^2}{2x-1}-\frac{4x^3}{4x^2-1}\right)$

   (c) $\displaystyle\lim_{x\to 0}\frac{\tan x-\sin x}{x^2}$

6. Determine whether these improper integrals converge or diverge: if an integral converges, give its value.

   (a) $\displaystyle\int_{-1}^{\infty}\frac{x\,dx}{(x^2+4)^2}$

   (b) $\displaystyle\int_0^2\frac{3\,dx}{(x-1)^2}$

7. Express $0.\overline{09} = 0.090909\ldots$ as a geometric series, and so express its value as a fraction in lowest terms.

8. Classify each of the following series as divergent, or convergent. Indicate which test(s) used and explain your logic briefly.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{2n}{5n-1}$

   (b) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{5\sin^2 n}{n^2}$

   (d) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+3)}}$

9. Classify each of the following series as divergent, absolutely convergent, or conditionally convergent. Again, state which tests are used and justify your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$

   (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n(n+2)!}{3^n n!}$

10. Find the interval of convergence for the power series

    $$\sum_{n=1}^{\infty}\frac{n(x-3)^n}{2^n}.$$

11. (a) Find the first 4 non-zero terms of the Taylor series for $f(x) = \ln x$ expanded around $x = 1$.

    (b) Find an expression for the n^{th} term of the series. Write the series in sigma notation.