CEGEP JOHN ABBOTT COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/∞)

1. Differentiate and simplify: $y = x\arcsin(2x) + \frac{1}{2}\sqrt{1-4x^2}$.

2. Evaluate the following six integrals.

   (a) $\displaystyle\int (2x+1)\sqrt{x^2+x+1}\,dx$ (b) $\displaystyle\int \sin^4 x\cos^3 x\,dx$

   (c) $\displaystyle\int x^2\sin 2x\,dx$ (d) $\displaystyle\int \frac{e^{2x}\,dx}{9+e^{4x}}$

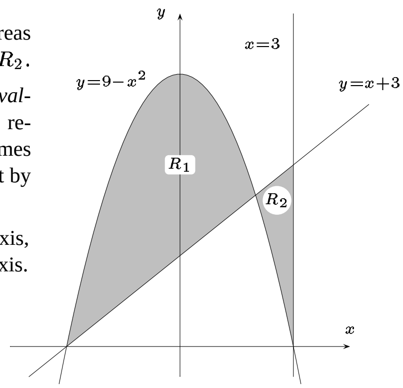
   (e) $\displaystyle\int_0^2 \frac{x^2\,dx}{\sqrt{4-x^2}}$ (f) $\displaystyle\int \frac{x^2-x+4}{(x+3)(x-1)^2}\,dx$

3. (a) Find the sum of the areas of the regions R_1 and R_2.

   (b) Set up — *but do not evaluate* — the integrals required to find the volumes of the solids that result by revolving

   (i) R_1 about the x-axis,

   (ii) R_2 about the y-axis.



4. Evaluate the following limits.

   (a) $\displaystyle\lim_{x\to 0^+}\sin x\ln x$ (b) $\displaystyle\lim_{x\to 0^+}(1+3x)^{\cot x}$ (c) $\displaystyle\lim_{x\to 1^+}\left(\frac{x}{\ln x}-\frac{3}{x-1}\right)$

5. Determine if the improper integral diverges or converges; if it converges, give its value.

   (a) $\displaystyle\int_1^5 \frac{dx}{x(\ln x)^2}$ (b) $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{9+x^2}$

6. Determine if the series is convergent or divergent. Name the test used. Verify the conditions of the test.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{(n+1)!}$ (b) $\displaystyle\sum_{n=1}^{\infty}\ln\left(\frac{2n+1}{3n+2}\right)$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{-\sqrt{n}}}{\sqrt{n}}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{2n-1}{\sqrt{n^3+4}}$

7. Classify each series as divergent, or absolutely convergent, or conditionally convergetn. Name the test used. Verify the conditions of the test.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\ln n}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{3n^2+1}$

8. For the series

$$\sum_{n=1}^{\infty}\frac{1}{3}\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right),$$

   let $\{S_n\}$ be the sequence of its partial sums.

   (a) Find $S_1,\ S_2,\ S_3,\ S_4$. (b) Find S_n. (c) Find the sum of the series.

9. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{(x+2)^n}{n3^n}.$$

10. (a) (i) Find the Taylor series for $f(x)=\sin x$ around $x=0$.

   (ii) Write the series in Σ-notation.

   (iii) Find the interval of convergence.

   (b) Using the series found in (a), expand $\sin\left(x^2\right)$ around $x=0$.

ANSWERS

1. $\arcsin 2x$

2. (a) $\frac{2}{3}\left(x^2+x+1\right)^{3/2}+C$

   (b) $\frac{1}{5}\sin^5 x-\frac{1}{7}\sin^7 x+C$

   (c) $-\frac{1}{2}x^2\cos 2x+\frac{1}{4}\cos 2x+\frac{1}{2}x\sin 2x+C$

   (d) $\frac{1}{6}\arctan\left(\frac{1}{3}e^{2x}\right)+C$

   (e) π

   (f) $\ln|x+3|-\dfrac{1}{x-1}+C$

3. (a) The area is $71/3$.

   (b) (i) $\pi\int_{-3}^{2}(9-x^2)^2-(x+3)^2\,dx$

   (ii) $2\pi\int_2^3 x(x^2+x-6)\,dx$

4. (a) 0 (b) e^3 (c) $-\infty$

5. (a) The integral diverges (to $-\infty$).

   (b) The integral converges to $\pi/3$.

6. (a) Convergent (RaT)

   (b) Divergent (the terms tend to $\ln\frac{2}{3}\neq 0$)

   (c) Convergent ($\int$-test)

   (d) Divergent (lim. comp. with $\sum n^{-1/2}$)

7. (a) Cond. conv. (comp. with, *e.g.*, $\sum n^{-1}$ & AST)

   (b) Abs. conv. (comp. with $\sum n^{-2}$)

8. (a) $\frac{1}{10},\ \frac{1}{8},\ \frac{3}{22},\ \frac{1}{7},\ \ldots,$

   (b) $S_n=\dfrac{1}{6}-\dfrac{1}{3(3n+2)}$

   (c) $1/6$

9. $[-5,1)$

10. (a) $\sin x = x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\dfrac{x^9}{9!}-\cdots$ converges for all real numbers (RaT).

   (b) $\sin\left(x^2\right)=x^2-\dfrac{x^6}{3!}+\dfrac{x^{10}}{5!}-\dfrac{x^{14}}{7!}+\cdots$