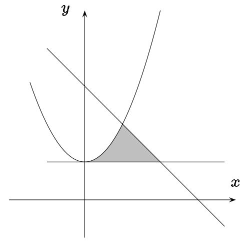
CEGEP JOHN ABBOTT COLLEGE

CALCULUS II (SCIENCE)

(MATHEMATICS 201–NYB/∞)

1. Let R be the region in quadrant I bounded by
$$y = x^2 + 1, \quad y = 1, \quad x + y = 3.$$
   (a) Find the area of R.
   (b) Set up the integral required to find the volume of the solid that results from revolving R about the x-axis. *Do not actually evaluate the integral.*
   (c) Set up the integral required to find the volume of the solid that results from revolving R about the y-axis. *Do not actually evaluate the integral.*



2. (a) If $y = \operatorname{arcsec}(x^2 + 5)$ find y'.
   (b) Find all values of x that make the slope of the graph $y = \arctan 3x$ equal to 1.

3. Evaluate the following integrals.
   (a) $\displaystyle\int_1^e x^2 \ln x \, dx$ (b) $\displaystyle\int e^{-x} \cos 2x \, dx$
   (c) $\displaystyle\int_0^{3/2} \frac{x^2 \, dx}{\sqrt{9 - x^2}}$ (d) $\displaystyle\int \tan^3 x \sec^5 x \, dx$
   (e) $\displaystyle\int \frac{x^2 + 4x + 2}{(x+1)^2(x+2)} \, dx$ (f) $\displaystyle\int x\sqrt{2x^2 - 3} \, dx$

4. Calculate the following limits.
   (a) $\displaystyle\lim_{x\to\infty} x\left(e^{1/x} - 1\right)$ (b) $\displaystyle\lim_{x\to\infty}\left(1 + \frac{1}{2x}\right)^x$ (c) $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{\tan 4x}$

5. Determine whether these improper integrals converge or diverge; if an integral converges, give the exact value of the the integral.
   (a) $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^2}$ (b) $\displaystyle\int_{-2}^2 \frac{dx}{x^4}$

6. For the sequence $\{a_k\} = \left\{\dfrac{e^k}{k^{10}}\right\}$, determine whether or not it is convergent. (Justify your answer.)

7. Calculate (if possible) the sum of each of the following series.
   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{n(n+1)}$ (b) $\displaystyle\sum_{n=1}^\infty \frac{2}{5^n}$

8. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)
   (a) $\displaystyle\sum_{n=2}^\infty \frac{\ln(n^2)}{n}$ (b) $\displaystyle\sum_{n=1}^\infty \frac{n!}{1000^n}$
   (c) $\displaystyle\sum_{n=1}^\infty \frac{17}{n^2 - 11}$ (d) $\displaystyle\sum_{n=1}^\infty \frac{2^n}{(2+n)^n}$

9. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)
   (a) $\displaystyle\sum_{k=0}^\infty \frac{(-1)^k}{\sqrt{k+2}}$ (b) $\displaystyle\sum_{k=0}^\infty \sin\left(\frac{k\pi}{2}\right)$

10. Determine the interval of convergence of the series
$$\sum_{n=0}^\infty \frac{(x+1)^n}{3^{n+1}(n+1)^3}.$$

11. For the function $f(x) = \sin 2x$
   (a) find the first four terms of the Maclaurin series for $f(x)$,
   (b) find a formula for the general term, and express the series in Σ-notation.
   (c) What is the radius of convergence of this series?

ANSWERS

1. (a) $\int_1^2 \left(3 - y - \sqrt{y-1}\right) dy = \frac{5}{6}$
   (b) $2\pi \int_1^2 y\left(3 - y - \sqrt{y-1}\right) dy$
   (c) $\pi \int_1^2 (3-y)^2 - (y-1)\, dy$
2. (a) $y' = \dfrac{2x}{(x^2+5)\sqrt{(x^2+5)^2 - 1}}$
   (b) $x = \pm\frac{1}{3}\sqrt{2}$.
3. (a) $\frac{1}{9}(2e^3 + 1)$
   (b) $\dfrac{2\sin 2x - \cos 2x}{5e^x} + C$
   (c) $\frac{3}{4}\pi - \frac{9}{8}\sqrt{3}$
   (d) $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$
   (e) $\ln\dfrac{|x+1|^3}{(x+2)^2} + \dfrac{1}{x+1} + C$
   (f) $\frac{1}{6}(2x^2 - 3)^{3/2} + C$
4. (a) 1 (b) $\sqrt{e}$ (c) $\frac{3}{4}$
5. (a) The integral converges to 1.
   (b) The integral diverges (to ∞).
6. The sequence diverges (to ∞).
7. (a) The sum of the series is 1.
   (b) The sum of the series is $\frac{1}{2}$.
8. (a) Divergent (comp. with $\sum n^{-1}$, or $\int$-test)
   (b) Divergent (RaT)
   (c) Convergent (lim. comp. with $\sum n^{-2}$)
   (d) Convergent (RoT)
9. (a) Cond. conv. (lim. comp. with $\sum k^{-1/2}$ & AST)
   (b) Divergent (the terms do not approach zero as $n \to \infty$)
10. $[-4, 2]$
11. (a) $2x - \dfrac{8}{3!}x^3 + \dfrac{32}{5!}x^5 - \dfrac{128}{7!}x^7 + \cdots$
   (b) $\displaystyle\sum_{n=0}^\infty (-1)^n \frac{2^{2n+1}}{(2n+1)!} x^{2n+1}$
   (c) The radius of convergence is infinite.