CEGEP
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COLLEGE

CALCULUS II (SCIENCE)

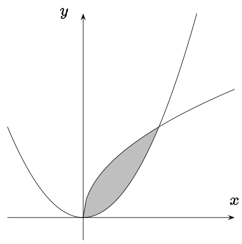
(MATHEMATICS 201–NYB/∞)

1. Let R be the region bounded by

   $y = \sqrt{8x}$ and $y = x^2$.

   (a) Find the area of R.

   (b) Find the volume of the solid of revolution obtained by rotating R about the y-axis.



2. For the following functions, calculate $\dfrac{dy}{dx}$.

   (a) $y = x \arctan(x^2 - 1)$ (b) $y = \dfrac{\arcsin x}{\sin x}$

3. Evaluate the following integrals.

   (a) $\displaystyle\int_0^2 \frac{x-1}{x^2+4}\,dx$ (b) $\displaystyle\int \tan^3 2x\,dx$

   (c) $\displaystyle\int_0^{\pi/2} \sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)dx$ (d) $\displaystyle\int e^{-2x}\sin 3x\,dx$

   (e) $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\,dx$ (f) $\displaystyle\int \frac{4x}{(x-1)^2(x+1)}\,dx$

4. Calculate the following limits.

   (a) $\displaystyle\lim_{x\to\pi} \frac{\sin^2 x}{1+\cos 3x}$ (b) $\displaystyle\lim_{x\to\infty} x^2 \sin\left(\frac{1}{x}\right)$ (c) $\displaystyle\lim_{x\to 0+} (1+2x)^{3/x}$

5. Determine whether these improper integrals converge or diverge: if an integral converges, give the exact value of the integral.

   (a) $\displaystyle\int_2^4 \frac{dx}{\sqrt{x-2}}$ (b) $\displaystyle\int_0^\infty \frac{x\,dx}{x^2+9}$

6. For the sequence $\{a_k\} = \left\{\dfrac{2k^4 - k^2 + 4}{3k(k^3+1)}\right\}$, determine whether or not it is convergent. (Justify your answer.)

7. Calculate (if possible) the sum of each of the following series.

   (a) $\displaystyle\sum_{n=2}^\infty \frac{1}{(n-1)(n+1)}$ (b) $\displaystyle\sum_{n=0}^\infty \frac{2+5^n}{7^n}$

8. Classify each of the following series as convergent or divergent. (Briefly justify your conclusions.)

   (a) $\displaystyle\sum_{n=1}^\infty \frac{2n^4 - n^2 + 4}{3n(n^3+1)}$ (b) $\displaystyle\sum_{n=0}^\infty \frac{2^n (n+2)!}{(3n)!}$

   (c) $\displaystyle\sum_{n=1}^\infty \left(\frac{2n^3 - 15}{5n^3 + n^2}\right)^n$ (d) $\displaystyle\sum_{n=1}^\infty \frac{\sqrt{n+1}}{n^3+3}$

9. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Briefly justify your conclusions.)

   (a) $\displaystyle\sum_{k=2}^\infty \frac{(-1)^k}{k \ln k}$ (b) $\displaystyle\sum_{k=1}^\infty \frac{\sin(k)}{k^2}$

10. Determine the interval of convergence of the series

$$\sum_{n=0}^\infty \frac{(x-2)^n}{5^n\sqrt{n+1}}.$$

11. For the function $f(x) = e^{2x}$

   (a) find the first four terms of the Maclaurin series for $f(x)$,
   (b) find a formula for the general term, and express the series in Σ-notation.
   (c) What is the radius of convergence of this series?

ANSWERS

1. (a) $8/3$ (b) $24\pi/5$
2. (a) $\arctan(x^2-1) + \dfrac{2x^2}{(x^2-1)^2+1}$
   (b) $\dfrac{\sin x - \cos x \arcsin x\sqrt{1-x^2}}{\sin^2 x\sqrt{1-x^2}}$
3. (a) $\frac{1}{2}\ln 2 - \pi/8$
   (b) $\frac{1}{4}\sec^2 2x - \frac{1}{2}\ln|\sec 2x| + C$
   (c) $\pi/16$
   (d) $-\dfrac{3\cos 3x + 2\sin 3x}{13e^{2x}} + C$
   (e) $-\frac{1}{x}\sqrt{9-x^2} - \arcsin\left(\frac{x}{3}\right) + C$
   (f) $\ln\left|\dfrac{x-1}{x+1}\right| - \dfrac{2}{x-1} + C$
4. (a) $2/9$ (b) ∞ (c) e^6
5. (a) The integral converges to $2\sqrt{2}$.
   (b) The integral diverges to ∞.
6. The sequence converges to $2/3$.
7. (a) The sum of the series is $3/4$.
   (b) The sum of the series is $35/6$.
8. (a) Divergent (the terms tend to $\frac{2}{3} \neq 0$)
   (b) Convergent (RaT)
   (c) Convergent (RoT)
   (d) Convergent (lim. comp. with $\sum n^{-5/2}$)
9. (a) Cond. conv. ($\int$-test & AST)
   (b) Abs. conv. (comp. with $\sum k^{-2}$)
10. The interval of convergence is $[-3, 7)$
11. (a) $1 + 2x + \dfrac{2^2}{2!}x^2 + \dfrac{2^3}{3!}x^3 + \cdots$
   (b) $\displaystyle\sum_{n=0}^\infty \frac{2^n}{n!}x^n$
   (c) The radius of convergence is infinite.