Examinations
from previous terms

CEGEP
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LINEAR ALGEBRA

(MATHEMATICS 201-NYC-05/∞)

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1. Given $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.
   (a) Find conditions on $a,\ b,\ c,\ d$ so that $AB = BA$.
   (b) Give an example of a matrix A such that $AB = BA$.

2. Given $C = \begin{pmatrix} 2 & -1 \\ 8 & -3 \end{pmatrix}$.
   (a) Find C^{-1}
   (b) If $FC = C^T$ find F.

3. Given $A^{-1} = \begin{pmatrix} 4 & 1 & -2 \\ 0 & 2 & 6 \\ 1 & -1 & 0 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 3 & -6 & 4 \\ -2 & 6 & -3 \\ 1 & -3 & 2 \end{pmatrix}$
   (a) Find $(3A^{-1} - 2B^{-1})$
   (b) Solve $A\vec{x} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$
   (c) Find $(AB)^{-1}$
   (d) Find $(2A)^{-1}$

4. Use

$$A = \begin{pmatrix} 1 & 5 & 0 \\ -2 & -7 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   to write
   (i) A^{-1} as a product of elementary matrices.
   (ii) A as a product of elementary matrices.

5. Use the LU solution method to solve $(LU)\vec{x} = \begin{pmatrix} 9 \\ 2 \\ 3 \end{pmatrix}$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -\frac{3}{2} & 1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} 3 & 4 & -1 \\ 0 & -4 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

   Note: No marks will be given if you do not use the LU method.

6. Given the system of equations

$$\begin{aligned} x + 3y - 4z + w &= 3 \\ 2x + 3y - z + 2x &= -3 \\ 4x + 9y - 9z + 4w &= 4. \end{aligned}$$

   (i) Find a general solution.
   (ii) Find numbers a and b so that $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} a \\ b \\ 0 \\ 1 \end{pmatrix}$ is a particular solution.

7. Find the inverse, if possible, of $A = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 2 & 3 \\ 3 & 0 & 1 \end{pmatrix}$.

8. Given $A(3, -3, -1)$, $B(2, -1, 3)$ and $C(-1, 2, 1)$.
   (a) Plot these points in a Cartesian system for $\mathbb{R}^3$ and show the triangle $\triangle ABC$.
   (b) Find the area (exact value) of $\triangle ABC$.
   (c) Find an equation for the plane containing $\triangle ABC$.

9. Let $\vec{u} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$.
   (i) Find $\vec{u} \times \vec{w}$.
   (ii) Find the projection vector of $\vec{v}$ onto $\vec{u}$ and the component of $\vec{v}$ orthogonal to $\vec{u}$.
   (iii) Compute $||2\vec{u} - \vec{v} + \vec{w}||$
   (iv) Find the volume of the parallelepiped with sides $\vec{u},\ \vec{v},\ \vec{w}$.

10. Gven the point $P_0(2, 8, 5)$ and the plane $x - 2y - 2z = 1$.
   (i) Find the distance from the point P_0 to the plane.
   (ii) Find the line containing the point P_0 and perpendicular to the given plane.
   (iii) Find the coordinates of the point on the given plane that is nearest to P_0.

11. Given planes $x + y + z = 1$ and $x - 2y + 3x = 1$.
   (i) Find the angle between the two planes (in degrees, 3 decimal place accuracy).
   (ii) Find the line of intersection of the two planes.

12. Let $A = \begin{pmatrix} 2 & 0 & -2 & 3 \\ 0 & -2 & -3 & 0 \\ -2 & 4 & 0 & 3 \\ 0 & 2 & -2 & 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, and $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$
   (a) Evaluate $\det(A)$.
   (b) Solve $A\vec{x} = \vec{b}$ for x_1 using Cramer's rule.
   (c) Compute (i) $det(2A)$, (ii) $\det(A^T A)$.

13. Given $\vec{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{u}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{u}_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.
   (a) Is $\vec{u}_4$ a linear combination of $\vec{u}_1,\ \vec{u}_2,\ \vec{u}_3$? Justify.
   (b) Are the vectors $\vec{u}_1,\ \vec{u}_2,\ \vec{u}_3$ and $\vec{u}_4$ linearly independent? Justify.

14. Let $\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ and $\vec{w}_1 = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$, $\vec{w}_2 = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$.
   (a) Is $\mathrm{Span}\{\vec{u}_1\} = \mathrm{Span}\{\vec{w}_1\}$? Justify.
   (b) Find an equation for $\mathrm{Span}\{\vec{u}_1,\ \vec{u}_2\}$.
   (c) Is $\mathrm{Span}\{\vec{u}_1,\ \vec{u}_2\} = \mathrm{Span}\{\vec{w}_1,\ \vec{w}_2\}$? Justify.

15. Given $A = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 4 \\ 1 & 2 & 3 & 3 \end{pmatrix}$, find a basis for
   (a) $\mathrm{Col}(A)$
   (b) $\mathrm{Row}(A)$
   (c) $\mathrm{Nul}(A)$
   (d) $\mathrm{Nul}(A^T)$

16. Let $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x \geqslant y \geqslant z \right\}$.
   (i) Is the zero vector in S?
   (ii) For what value(s) of k, if any, is $\begin{pmatrix} 2 \\ k \\ 3 \end{pmatrix}$ in S?
   (iii) Is S closed under addition?
   (iv) Is S closed under scalar multiplication?
   (v) Is S a subspace of $\mathbb{R}^3$?

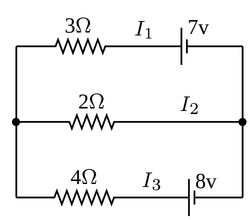
17. Let $V = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : x_1 + 2x_2 - x_3 = 0 \right\}$.
   (a) Find a basis for the susbspace V.
   (b) Which of the following are true? Justify.
      (i) V is a 2-dimensional subspace of $\mathbb{R}^3$.
      (ii) V is a 3-dimensional subspace of $\mathbb{R}^3$.
      (iii) $V = \mathrm{Nul}(A)$, where $A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \end{pmatrix}$.

18. Balance the following chemical equation:

$$C_7H_8 + HNO_3 \leftrightharpoons C_7H_5O_6N_3 + H_2O$$

   Define all variables used. Set up a system of equations and use the augmented matrix to solve the system. State your final answer.

19. Set up, but *do not solve*, an appropriate system of equations for the currents $I_1,\ I_2,\ I_3$ for the electrical circuit.



20. Let $\vec{u}_1,\ \vec{u}_2$ and $\vec{u}_3$ be linearly independent vectors in $\mathbb{R}^3$. Prove that $\{\vec{u}_1,\ \vec{u}_2,\ \vec{u}_3\}$ is a basis for $\mathbb{R}^3$.