1. Find a quadratic polynomial p such that $p(1) = 1$, $p(2) = 6$ and $p'(2) = 6$.

2. Consider the following linear system.

$$\begin{array}{rcrcrcl} 2x & + & 3y & + & 4z & = & 3 \\ 2x & + & (h+1)y & + & 6z & = & 4 \\ 4x & + & 6y & + & (h-4)z & = & k-1 \end{array}$$

Find all values of h and k such that the system has: a. a unique solution; b. infinitely many solutions; c. no solution.

3. Consider the following matrix A and its reduced echelon form R.

$$A = \begin{pmatrix} 2 & 1 & -1 & 1 & 0 & 3 \\ 3 & 4 & -14 & 1 & 0 & 14 \\ 1 & -1 & 7 & 2 & 0 & -9 \\ -3 & -2 & 4 & 3 & 0 & -24 \end{pmatrix} \qquad R = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 & 2 \\ 0 & 1 & -5 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

a. Find a basis of $\mathrm{Col}(A)$, $\mathrm{Col}(A^T)$, $\mathrm{Nul}(A)$.

b. Determine the dimension of $\mathrm{Nul}(A^T)$.

c. Express the fourth column of A as a linear combination of the other columns of A.

d. Find a vector in $\mathrm{Nul}(A)$ whose first and second entries are 18 and -5.

e. Given that $\mathbf{e}_1+2\mathbf{e}_2+3\mathbf{e}_3+4\mathbf{e}_4+5\mathbf{e}_5+6\mathbf{e}_6$ is a solution of $A\mathbf{x} = \mathbf{b}$, determine the general solution to $A\mathbf{x} = \mathbf{b}$.

4. Let $A = \begin{pmatrix} k & 1 & 2 \\ 1 & k & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
a. Compute $\det(A)$.
b. Find all values of k for which A is singular.

5. Let $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ reflect vectors in the x-axis and let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the horizontal shear such that $T(\mathbf{e}_1 + \mathbf{e}_2) = 3\mathbf{e}_1 + \mathbf{e}_2$.
a. Find the standard matrix of S. b. Find the standard matrix of T.
c. Find the standard matrix of $S \circ T$.
d. Find a non-zero vector $\mathbf{v} \in \mathbb{R}^2$ such that $S(\mathbf{v}) = T(\mathbf{v})$.

6. Let $A = \begin{pmatrix} 2 & 4 \\ 2 & 6 \\ -1 & 6 \\ 0 & -6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -6 \\ -14 \\ -29 \\ 24 \end{pmatrix}$. Find an LU-factorization of A, and use it to solve $A\mathbf{x} = \mathbf{b}$.

7. Express the matrix $\begin{pmatrix} 2 & 4 \\ 2 & 3 \end{pmatrix}$ as a product of elementary matrices.

8. Given $A = \begin{pmatrix} -2 & 6 \\ 4 & -7 \end{pmatrix}$, find a matrix X such that $XA - XA^T = A$.

9. Let A and B be 4×4 matrices.
a. If $A^TA = 2I$, what are the possible values of $\det(A)$?
b. If $\det(B) = -2$, what is the value of $\det\big((B^{-1})^3 \operatorname{adj}(B)\big)$?

10. Let A be a 3×3 matrix such that

$$A\begin{pmatrix} 4 \\ 6 \\ 9 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \qquad A\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} \qquad \text{and} \qquad A\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

a. Find a vector $\mathbf{u}$ such that $A\mathbf{u} = \mathbf{e}_2$. b. Determine A^{-1}.

11. A matrix A is called *skew-symmetric* if $A^T = -A$. Show that if A and B are skew-symmetric $n \times n$ matrices, and $AB = -BA$, then AB is skew-symmetric.

12. Consider a matrix A such that:
- A is in reduced echelon form;
- the first two columns of A are linearly independent;
- the null space of A is in $\mathbb{R}^5$;
- the span of the fourth column of A is a singleton subset of $\mathbb{R}^3$;
- the dimension of the null space of A is greater than the rank of A.

a. What is the size of A?

b. Give an example of a matrix which satisfies all of the conditions above.

13. Given that A is a square matrix and

$$\begin{pmatrix} W & 0 \\ X & Y \\ Z & I \end{pmatrix}\begin{pmatrix} A & 0 \\ B & I \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & A \\ 0 & I \end{pmatrix},$$

express W, X, Y and Z in terms of A and B.

14. The linear transformation $T\colon \mathbb{P}_2 \to \mathbb{R}^2$ is defined by $T(p) = \begin{pmatrix} p(1) \\ p'(0) \end{pmatrix}$.
a. Find the image of $x + x^2$ under T.
b. Find a polynomial $r(x)$ such that $T(r(x)) = 5\mathbf{e}_1 + 2\mathbf{e}_2$.
c. Find a basis for the kernel of T.
d. Is T injective? Justify your answer.

15. Let $\mathscr{H} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x(2y - 3z) = 0 \right\}$.
a. Is $\mathscr{H}$ closed under vector addition?
b. Is $\mathscr{H}$ closed under scalar multiplication?
c. Is $\mathscr{H}$ a subspace of $\mathbb{R}^3$?

16. Find a basis of $\mathscr{H} = \left\{ \mathbf{x} \in \mathbb{R}^4 : \mathbf{x} \cdot \begin{pmatrix} 1 \\ -3 \\ 0 \\ 2 \end{pmatrix} = 0 \right\}$.

17. Let $p(x) = 2 + 3x - x^2$, $q(x) = -3 - 5x + 3x^2$ and $r(x) = 4 + 3x + 7x^2$. Show that $r(x) \in \mathrm{Span}\{p(x), q(x)\}$.

18. You are given the planes $\wp_1 : 2x - y + 5z = 8$, $\wp_2 : x - y + 2z = 4$, and the point $A(5, -3, 2)$.
a. Find the cosine of the angle between $\wp_1$ and $\wp_2$.
b. Find the point on the plane $\wp_1$ which is closest to A.
c. Find an equation of the line which contains A and is parallel to both $\wp_1$ and $\wp_2$.

19. Given the line

$$\ell\colon \mathbf{x} = \begin{pmatrix} k \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} h \\ 1 \\ -2 \end{pmatrix},$$

and the plane $\mathscr{P}\colon x - 2y + 4z = 0$, find conditions on h and k for each of the following cases:
a. ℓ and $\mathscr{P}$ are parallel and do not intersect;
b. ℓ is contained in $\mathscr{P}$; c. ℓ and $\mathscr{P}$ intersect at a point.

20. Given the points $P(0,0,-2)$, $Q(2,3,4)$, $R(4,6,5)$ and $S(6,11,10)$, find the following.
a. The area of triangle PQR.
b. A normal equation of the plane containing P, Q and R.
c. The volume of the parallelepiped with edges $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$.
d. A point on the line through P and Q which is two units away from P.

21. Show that if $\mathrm{proj}_{\mathbf{v}}\,\mathbf{u} = \mathrm{proj}_{\mathbf{v}}\,\mathbf{w}$ then $\mathbf{u} - \mathbf{w}$ is orthogonal to $\mathbf{v}$.

22. Complete the following sentences with the words **must**, **might** or **cannot** as appropriate.
a. If $\mathbf{u} \in \mathrm{Span}\{\mathbf{v}\}$, then $\mathbf{v}$ __________ belong to $\mathrm{Span}\{\mathbf{u}\}$.
b. If A is invertible, then $\mathrm{Row}(A)$ __________ be identical to $\mathrm{Col}(A)$.
c. If A is a 3×3 matrix of rank 2 and $\mathbf{b} = \mathbf{e}_3 \in \mathbb{R}^3$, then the equation $A\mathbf{x} = \mathbf{b}$ __________ be inconsistent.
d. If A, B and C are square matrices such that $ABC^2 = I$ then C __________ be invertible.

1. If p is quadratic and $p(2) = p'(2) = 6$, then there is a real number α such that $p(x) = 6 + 6(x-2) + \alpha(x-2)^2$, and the condition $p(1) = 1$ yields $\alpha = 1$. Thus, $p(x) = 6 + 6(x-2) + (x-2)^2 = x^2 + 2x - 2$.

2. The augmented matrix of the system is

$$\begin{pmatrix} 2 & 3 & 4 & 3 \\ 2 & h+1 & 6 & 4 \\ 4 & 6 & h-4 & k-1 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 & 4 & 3 \\ 0 & h-2 & 2 & 1 \\ 0 & 0 & h-12 & k-7 \end{pmatrix}.$$

The system has a unique solution if $h \neq 2, 12$ and k is any real number. The system has infinitely many solutions if $h = 12$ and $k = 7$, or if $h = 2$ and $k = 2$. The system has no solution if $h - 12$ and $k \neq 7$, or if $h = 2$ and $k \neq 2$.

3. Write $\mathbf{a}_j$ for column j of A, and write $\mathbf{r}_j$ for column j of R^T.

a. $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_4\}$ is a basis of $\mathrm{Col}(A)$, $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\}$ is a basis of $\mathrm{Col}(A^T)$, and

$$\left\{ \begin{pmatrix} -2 \\ 5 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ 0 \\ 4 \\ 0 \\ 1 \end{pmatrix} \right\}$$

is a basis of $\mathrm{Nul}(A)$.

b. The dimension of $\mathrm{Nul}(A^T)$ is 1.

c. From the third given basis vector of $\mathrm{Nul}(A)$, $\mathbf{a}_4 = \frac{1}{2}\mathbf{a}_1 + \frac{3}{4}\mathbf{a}_2 - \frac{1}{4}\mathbf{a}_6$.

d. It suffices to solve $a + b = -9$ and $5a - 3b = -5$, or $8a = -32$, so $a = -4$ and $b = -5$, giving the vector

$$\begin{pmatrix} 18 \\ -5 \\ -4 \\ -20 \\ 0 \\ -5 \end{pmatrix}$$

in $\mathrm{Nul}(A)$.

e. The general solution of $A\mathbf{x} = \mathbf{b}$ is $\mathbf{p} + \mathrm{Nul}(A)$, where $\mathbf{p}$ is the given solution.

4. The determinant of A is equal to

$$\begin{vmatrix} k-2 & -1 & 0 \\ 0 & k-1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (k-1)(k-2),$$

so A is singular if, and only if, $k = 1, 2$.

5. The standard matrices A of S and B of T are

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad \text{so} \quad AB = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

is the standard matrix of $S \circ T$. Any vector $\mathbf{x} \in \mathrm{Nul}(A - B) = \mathrm{Span}\{\mathbf{e}_1\}$ satisfies $S(\mathbf{x}) = T(\mathbf{x})$; so, *e.g.*, $\mathbf{e}_1$ is a non-zero vector as required.

6. The reduced second column is

$$\begin{pmatrix} 2 \\ 8 \\ -6 \end{pmatrix}, \quad \text{so} \quad A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -\frac{1}{2} & 4 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Solving $L\mathbf{y} = \mathbf{b}$ for $\mathbf{y}$ gives

$$y_1 = -6, \quad y_2 = -8, \quad y_3 = -29 - 3 + 32 = 0 \quad \text{and} \quad y_4 = 24 - 24 = 0.$$

Then, solving $U\mathbf{x} = \mathbf{y}$ for $\mathbf{x}$

$$x_2 = \tfrac{1}{2}(-8) = -4 \quad \text{and} \quad x_1 = \tfrac{1}{2}(-6 + 16) = 5.$$

7. From the reduction

$$\begin{pmatrix} 2 & 4 \\ 2 & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & 4 \\ 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \sim I_2,$$

it follows that

$$\begin{pmatrix} 2 & 4 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

8. Since

$$A - A^T = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

is nonsingular,

$$X = A(A - A^T)^{-1} = \frac{1}{4} \begin{pmatrix} -2 & 6 \\ 4 & -7 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 & 2 \\ -7 & -4 \end{pmatrix}.$$

9. a. Since $(\det A)^2 = \det(A^T A) = \det(2I) = 2^4$, it follows that $\det(A) = \pm 4$.

b. If $\det(B) = -2$ then $\det\big((B^{-1})^3 \operatorname{adj}(B)\big) = \left(-\frac{1}{2}\right)^3 (-2)^{4-1} = 1$.

10. From the given information $A(-3\mathbf{e}_1 - \mathbf{e}_3) = \mathbf{e}_2$, and

$$A^{-1} = \begin{pmatrix} \frac{4}{3} & -3 & 4 \\ 2 & 0 & 3 \\ 3 & -1 & 1 \end{pmatrix}.$$

11. If A, B are skew-symmetric $n \times n$ matrices and $AB = -BA$ then

$$(AB)^T = B^T A^T = (-B)(-A) = BA = -AB,$$

so AB is skew-symmetric.

12. The span of the columns of A is a subspace of $\mathbb{R}^3$ and the null space of A is a subspace of $\mathbb{R}^5$, so A is a 3×5 matrix. An example of a matrix satisfying all of the given conditions is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The first and second entries of the third and fourth columns of this matrix could be replaced by any real numbers, and the resulting matrix would still satisfy all of the given conditions.

13. The product on the left side of the given equation is

$$\begin{pmatrix} WA & 0 \\ XA + YB & Y \\ ZA + B & I \end{pmatrix},$$

so the given equation is equivalent to $WA = I$, $XA + YB = 0$ $ZA + B = 0$ and $Y = A$. Thus, $W = A^{-1}$, $X = -ABA^{-1}$, $Y = A$ and $Z = -BA^{-1}$.

14. a. $T(x + x^2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

b. The matrix of T relative to the standard bases of $\mathbb{P}_2$ and $\mathbb{R}^2$ is

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

so $1 + 2x + 2x^2$ belongs to the preimage under T of $5\mathbf{e}_1 + 2\mathbf{e}_2$.

c. A polynomial $p \in \ker(T)$ must have the form $\alpha + \beta x^2$ since $p'(0) = 0$; as well, $\alpha + \beta = 0$ since $p(1) = 0$. Thus, $\{1 - x^2\}$ is a basis of the kernel of T.

d. Since the kernel of T is not $\{0\}$, T is not injective. Notice that no linear transformation $\mathbb{P}_2 \to \mathbb{R}^2$ is injective, since $\dim(\mathbb{P}_2) = 3 > 2 = \dim(\mathbb{R}^2)$.

15. a. Since

$$\begin{pmatrix}1\\3\\2\end{pmatrix} \in \mathscr{H} \quad \text{and} \quad \begin{pmatrix}0\\-2\\-1\end{pmatrix} \in \mathscr{H}, \quad \text{but} \quad \begin{pmatrix}1\\3\\2\end{pmatrix} + \begin{pmatrix}0\\-2\\-1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} \notin \mathscr{H},$$

it follows that $\mathscr{H}$ is not closed under addition.

b. If $\alpha \in \mathbb{R}$ and

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \in \mathscr{H}, \quad \text{then} \quad \alpha\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}\alpha x\\\alpha y\\\alpha z\end{pmatrix} \in \mathscr{H},$$

since $(\alpha x)(2(\alpha y) - 3(\alpha z)) = \alpha^2 x(2y - 3z) = \alpha \cdot 0 = 0$. Therefore, $\mathscr{H}$ is closed under scalar multiplication.

c. Since $\mathscr{H}$ is not closed under addition, it is not a subspace of $\mathbb{R}^3$.

16. Notice that $\mathscr{H} = \text{Nul}(1 \ \ -3 \ \ 0 \ \ 2)$, a basis of which is

$$\left\{\begin{pmatrix}3\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}-2\\0\\0\\1\end{pmatrix}\right\}.$$

17. Since

$$\begin{pmatrix}2 & -3 & 4\\3 & -5 & 3\\-1 & 3 & 7\end{pmatrix} \sim \begin{pmatrix}2 & -3 & 4\\0 & -\frac{1}{2} & -3\\0 & \frac{3}{2} & 9\end{pmatrix} \sim \begin{pmatrix}2 & 0 & 22\\0 & 1 & 6\\0 & 0 & 0\end{pmatrix} \sim \begin{pmatrix}1 & 0 & 11\\0 & 1 & 6\\0 & 0 & 0\end{pmatrix},$$

it follows that $r(x) = 11p(x) + 6q(x)$.

18. a. Writing $\mathbf{n}_j$ for the given normal to $\wp_j$, the angle ϑ between $\wp_1$ and $\wp_2$ satisfies

$$\cos(\vartheta) = \frac{|\mathbf{n}_1{}^T\mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{13}{\sqrt{30}\sqrt{6}} = \tfrac{13}{30}\sqrt{5}.$$

b. Taking $B(4,0,0) \in \wp_1$ and $\mathbf{u} = \overrightarrow{AB}$, the point on $\wp_1$ which is closest to A is

$$\overrightarrow{OA} + \frac{\mathbf{u}^T\mathbf{n}_1}{\mathbf{n}_1{}^T\mathbf{n}_1}\mathbf{n}_1 = \begin{pmatrix}5\\-3\\2\end{pmatrix} - \tfrac{15}{30}\begin{pmatrix}2\\-1\\-5\end{pmatrix} = \tfrac{1}{2}\begin{pmatrix}8\\-5\\7\end{pmatrix}.$$

c. Since

$$\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix}2\\-1\\5\end{pmatrix} \times \begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}3\\1\\-1\end{pmatrix},$$

the line which is parallel to both $\wp_1$ and $\wp_2$ and contains A is

$$\overrightarrow{OA} + \text{Span}\{\mathbf{n}\} = \begin{pmatrix}5\\-3\\2\end{pmatrix} + \text{Span}\left\{\begin{pmatrix}3\\1\\-1\end{pmatrix}\right\}.$$

19. a. The direction of ℓ is parallel to the plane $\mathscr{P}$ if $h - 2 - 8 = 0$, or $h = 10$, and ℓ does not intersect $\mathscr{P}$ if, in addition, $k - 4 \neq 0$, or $k \neq 4$.

b. The line ℓ is contained in the plane $\mathscr{P}$ if $h = 10$ and $k = 4$.

c. The line ℓ meets the plane $\mathscr{P}$ in one point if $h \neq 10$ and k is any real number.

20. Let

$$\mathbf{u} = \overrightarrow{PQ} = \begin{pmatrix}2\\3\\6\end{pmatrix}, \quad \mathbf{v} = \overrightarrow{PR} = \begin{pmatrix}4\\6\\7\end{pmatrix} \quad \text{and} \quad \mathbf{w} = \overrightarrow{PS} = \begin{pmatrix}6\\11\\12\end{pmatrix}.$$

a. The area of $\triangle PQR$ is half the length of

$$\mathbf{u} \times \mathbf{v} = \begin{pmatrix}-15\\10\\0\end{pmatrix} = -5\begin{pmatrix}3\\-2\\0\end{pmatrix} = -5\mathbf{n},$$

or $\frac{5}{2}\|\mathbf{n}\| = \frac{5}{2}\sqrt{13}$.

b. The plane containing P, Q and R is defined by $\mathbf{n}^T\mathbf{x} = \mathbf{n}^T\overrightarrow{OP}$, or $3x - 2y = 0$.

c. The volume of the parallelpiped is $|\det(\mathbf{u} \ \ \mathbf{v} \ \ \mathbf{w})| = 5|\mathbf{n}^T\mathbf{w}| = 5|-4| = 20$.

d. The two such points are

$$\overrightarrow{OP} \pm \frac{2}{\|\mathbf{u}\|}\mathbf{u} = \begin{pmatrix}0\\0\\-2\end{pmatrix} \pm \tfrac{2}{7}\begin{pmatrix}2\\3\\6\end{pmatrix}; \quad \textit{i.e.}, \quad \tfrac{2}{7}\begin{pmatrix}2\\3\\-1\end{pmatrix} \quad \text{and} \quad \tfrac{2}{7}\begin{pmatrix}-2\\-3\\-13\end{pmatrix}.$$

21. As $\text{proj}_{\mathbf{v}}$ is a linear transformation, $\text{proj}_{\mathbf{v}}(\mathbf{u} - \mathbf{w}) = \text{proj}_{\mathbf{v}}\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{w} = \mathbf{0}$, so $\text{perp}_{\mathbf{v}}(\mathbf{u} - \mathbf{w}) = \mathbf{u} - \mathbf{w} - \text{proj}_{\mathbf{v}}(\mathbf{u} - \mathbf{w}) = \mathbf{u} - \mathbf{w}$; thus $\mathbf{u} - \mathbf{w}$ is orthogonal to $\mathbf{v}$.

22. Complete the following sentences with the words **must**, **might** or **cannot** as appropriate.

a. If $\mathbf{u} \in \text{Span}\{\mathbf{v}\}$, then $\mathbf{v}$ **might** belong to $\text{Span}\{\mathbf{u}\}$.

b. If A is invertible, then $\text{Row}(A)$ **must** be identical to $\text{Col}(A)$.

c. If A is a 3×3 of rank 2 and $\mathbf{b} = \mathbf{e}_3 \in \mathbb{R}^3$, then the equation $A\mathbf{x} = \mathbf{b}$ **might** be inconsistent.

d. If A, B and C are square matrices such that $ABC^2 = I$ then C **must** be invertible.